Why learn this?

You can use sums of sequences to find the size of a house of cards.(See Example4.)

Lesson Objective(s):

- Evaluate the sum of a series expressed in sigma notation.

You learned how to find the *n*th term of a sequence. Often we are also interested in the sum of a certain number of terms of a sequence. A series is the indicated sum of the terms of a sequence. Some examples are shown in the table.

Sequence	1, 2, 3, 4	2, 4, 6, 8, ...	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1 + 2 + 3 + 4$	$2 + 4 + 6 + 8 + \cdots$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

Because many sequences are infinite and do not have defined sums, we often find partial sums. A partial sum, indicated by S_n, is the sum of a specified number of terms of a sequence.

For the even numbers:

$S_1 = 2$	*Sum of first term*
$S_2 = 2 + 4 = 6$	*Sum of first 2 terms*
$S_3 = 2 + 4 + 6 = 12$	*Sum of first 3 terms*
$S_4 = 2 + 4 + 6 + 8 = 20$	*Sum of first 4 terms*

A series can also be represented by using summation notation, which uses the Greek letter $\sum$ (capital *sigma*) to denote the sum of a sequence defined by a rule, as shown.

$$\sum_{k=1}^{5} 2k$$

5 ← Last value of *k*

2*k* ← Explicit formula for sequence

k=1 ← First value of *k*

EXAMPLE 1

Using Summation Notation

Write each series in summation notation.

A $3 + 6 + 9 + 12 + 15$

B $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$

EXAMPLE 2

Evaluating a Series

Expand each series and evaluate.

A $\sum_{k=3}^{6} \frac{1}{2^k}$

B $\sum_{k=1}^{4} \left(10 - k^2\right)$

Finding the sum of a series with many terms can be tedious. You can derive formulas for the sums of some common series.

In a *constant series*, such as $3 + 3 + 3 + 3 + 3$, each term has the same value.

$$\sum_{k=1}^{5} 3 = \underbrace{3 + 3 + 3 + 3 + 3}_{\text{5 terms}} = 5 \cdot 3 = 15$$

The formula for the sum of a constant series is $\sum_{k=1}^{n} c = nc$, as shown.

$$\sum_{k=1}^{n} c = \underbrace{c + c + c + \cdots + c}_{n \text{ terms}} = nc$$

A *linear series* is a counting series, such as the sum of the first 10 natural numbers. Examine when the terms are rearranged.

$$\begin{aligned}\sum_{1}^{10} k &= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 \\ &= (1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6) \\ &= \underbrace{11 + 11 + 11 + 11 + 11}_{\text{5 terms}} = 5(11) = 55\end{aligned}$$

Notice that 5 is half of the number of terms and 11 represents the sum of the first and the last term, $1 + 10$. This suggests that the sum of a linear series is $\sum_{k=1}^{n} k = \frac{n}{2}(1 + n)$, which can be written as $\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$.

Similar methods will help you find the sum of a *quadratic series*.

Know it! Note

Summation Formulas

CONSTANT SERIES	LINEAR SERIES	QUADRATIC SERIES
$\sum_{k=1}^{n} c = nc$	$\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$	$\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$

EXAMPLE 3

Using Summation Formulas

Evaluate each series.

A $\sum_{k=5}^{10} 8$

B $\sum_{k=1}^{5} k$

C $\sum_{k=1}^{7} k^2$

EXAMPLE 4

Problem-Solving Application

Ricky is building a card house similar to the one shown. He wants the house to have as many stories as possible with a deck of 52 playing cards. How many stories will Ricky's house have?