Lesson Objective(s):

- Find the nth term of a sequence.
- Write rules for sequences.

In 1202, Italian mathematician Leonardo Fibonacci described how fast rabbits breed under ideal circumstances. Fibonacci noted the number of pairs of rabbits each month and formed a famous pattern called the *Fibonacci sequence*.

A sequence is an ordered set of numbers. Each number in the sequence is a term of the sequence. A sequence may be an infinite sequence that continues without end, such as the natural numbers, or a finite sequence that has a limited number of terms, such as {1, 2, 3, 4}.

You can think of a sequence as a function with sequential natural numbers as the domain and the terms of the sequence as the range. Values in the domain are called *term numbers* and are represented by *n*. Instead of function notation, such as *a(n)*, sequence values are written by using subscripts. The first term is a_1, the second term is a_2, and the *n*th term is a_n. Because a sequence is a function, each number *n* has only one term value associated with it, a_n.

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Reading Math

Term number	n	1	2	3	4	5	Domain
Term value	a_n	1	1	2	3	5	Range

In the Fibonacci sequence, the first two terms are 1 and each term after that is the sum of the two terms before it. This can be expressed by using the rule $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-2} + a_{n-1}$, where $n \geq 3$. This is a *recursive formula*. A recursive formula is a rule in which one or more previous terms are used to generate the next term.

EXAMPLE 1

Finding Terms of a Sequence by Using a Recursive Formula

Find the first 5 terms of the sequence with $a_1 = 5$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$.

n	$2a_{n-1} + 1$	a_n

In some sequences, you can find the value of a term when you do not know its preceding term. An explicit formula defines the nth term of a sequence as a function of n.

EXAMPLE 2

Finding Terms of a Sequence by Using an Explicit Formula

Find the first 5 terms of the sequence $a_n = 2^n - 3$.

n	$2^n - 3$	a_n

You can use your knowledge of functions to write rules for sequences.

EXAMPLE 3

Writing Rules for Sequences

Write a possible explicit rule for the nth term of each sequence.

A 3, 6, 12, 24, 48, ...

B 2.5, 4, 5.5, 7, 8.5, ...

EXAMPLE 4

Physics Application

A ball is dropped and bounces to a height of 5 feet. The ball rebounds to 60% of its previous height after each bounce. Graph the sequence and describe its pattern. How high does the ball bounce on its 9th bounce?

EXAMPLE 5

Iteration of Fractals

The Sierpinski triangle is a fractal made by taking an equilateral triangle, removing an equilateral triangle from the center, and repeating for each new triangle. Find the number of triangles in the next 2 iterations.