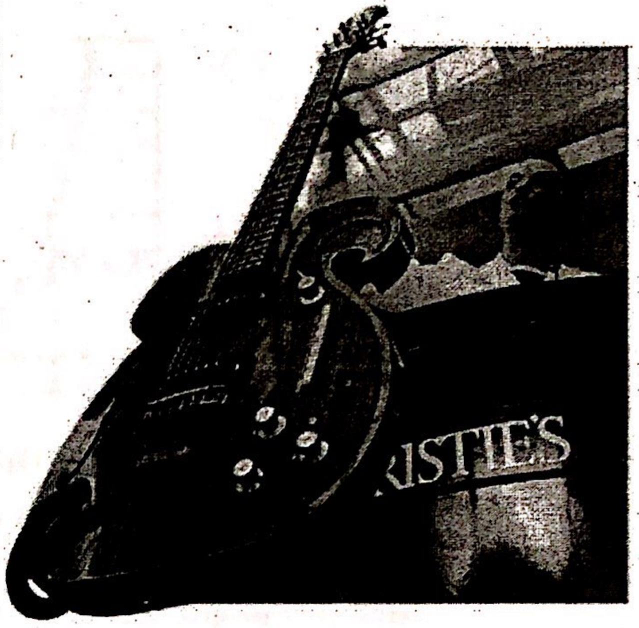
Exponential Functions, Growth, and Decay

Who uses this?

Collectors can use exponential functions to model the value of rare musical instruments.(See Example 2.)



Lesson Objective(s):

- Write and evaluate exponential expressions to model growth and decay situations.

Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

Transistors per Integrated Chip							
Year	1965	1966	1967	1968	1969	1970	1971
Transistors	60	120	240	480	960	1920	3840

×2 ×2 ×2 ×2 ×2 ×2

Growth that doubles every year can be modeled by using a function with a variable as an exponent. This function is known as an *exponential function*. The parent function is $y = b^x$, where b is a constant and the exponent *x* is the independent variable.

$$y = a \cdot b^x$$

a: initial amount

b: base, growth rate

$b > 1$ growth

$0 < b < 1$ decay

The graph of the parent function $f(x) = 2^x$ is shown. The domain is all real numbers and the range is $\{y|y > 0\}$.

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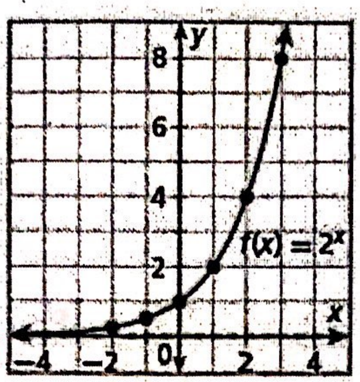
x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



as $x \to +\infty$, then $f(x) \to +\infty$

as $x \to -\infty$, then $f(x) \to 0$

Notice that as the x-values decrease, the graph of the function gets closer and closer to the x-axis. The function never reaches the x-axis because the value of 2^x cannot be zero. In this case, the x-axis is asymptote. An asymptote is a line that a graphed function approaches as the value of x gets very large or very small.

A function of the form $f(x) = ab^x$, with $a>0$ and $b>1$ is exponential growth function, which increases as x increases. When $0<b<1$, the function is called an exponential decay which decreases as x increases.

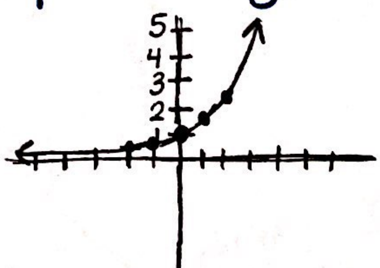
EXAMPLE 1

Graphing Exponential Functions

Tell whether the function shows growth or decay. Then graph.

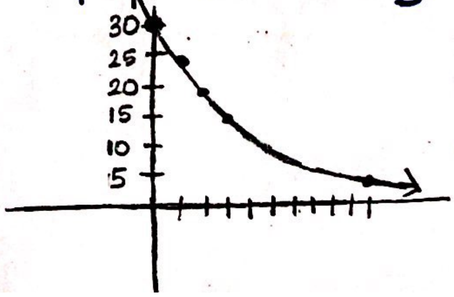
A $f(x) = 1.5^x$ exponential growth $b>1$

$a=1$



B $g(x) = 30(0.8^x)$ exponential decay $0<b<1$

$a=30$



EXAMPLE 2

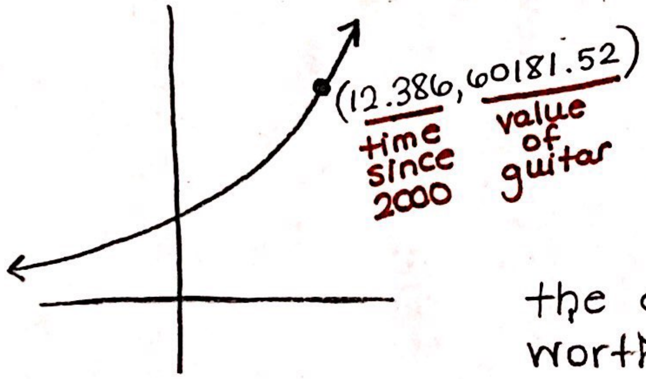
Economics Application

initial = a

Tony purchased a rare 1959 Gibson Les Paul guitar in 2000 for $12,000. Experts estimate that its value will increase by 14% per year. Use a graph to find when the value of the guitar will be $60,000.

help find b
1 ± %
1 + 0.14
1.14 = b

$y = 12000(1.14^x)$



the guitar will be worth $60,000 sometime in 2012

EXAMPLE 3

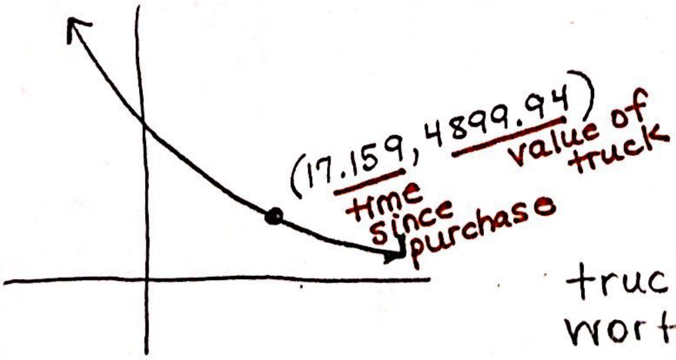
Depreciation Application

initial = a

1 ± %
1 − 0.095
0.905 = b

The value of a truck bought new for $28,000 decreases 9.5% each year. Write an exponential function, and graph the function. Use the graph to predict when the value will fall to $5000.

$y = 28000(0.905^x)$



truck will be worth $5000 in about 17 years after purchase