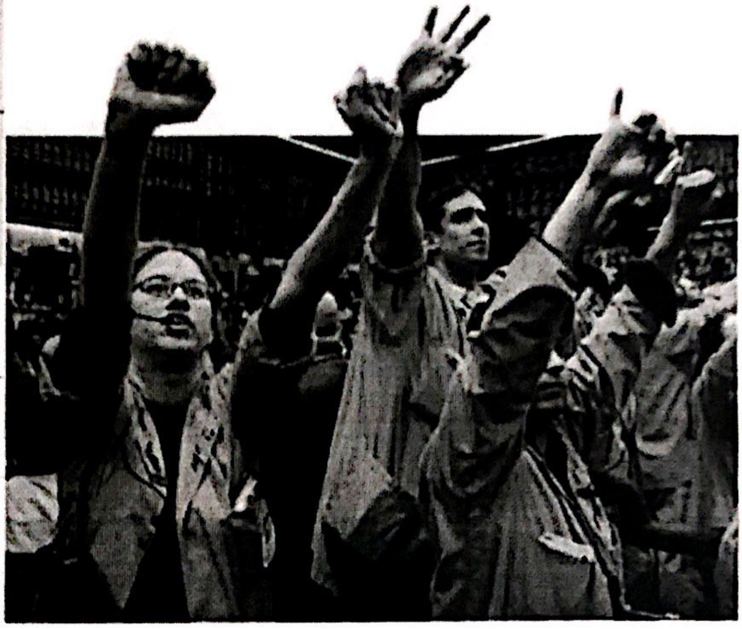
Curve Fitting with Polynomial Models

Who uses this?

Market analysts can use curve fitting to predict the performance of a stock index. (See Example 3.)



Lesson Objective(s):

- Use finite differences to determine the degree of a polynomial that will fit a given set of data.
- Use technology to find polynomial models for a given set of data.

The table shows the closing value of a stock index on the first day of trading for various years.

Year	1994	1995	1996	1997	2000	2001	2003	2004
Price ($)	774	751	1053	1293	4186	2474	1347	2011

−23 +302 +240

To create a mathematical model for the data, you will need to determine what type of function is most appropriate. You have learned that a set of data that has constant second differences can be modeled by a quadratic function. Finite differences can be used to identify the degree of any polynomial data. $r^2 \Rightarrow$ want to be close to 1

Finite Differences of Polynomials		
Function Type	**Degree**	**Constant Finite Differences**
Linear	1	first difference
Quadratic	2	second difference
Cubic	3	third difference
Quartic	4	fourth difference
Quintic	5	fifth difference

EXAMPLE 1

Using Finite Differences to Determine Degree

Use finite differences to determine the degree of the polynomial that best describes the data.

A

x	−2	−1	0	1	2	3
y	−10	−4	−1.4	0	2.4	8

+6 +2.6 +1.4 +2.4 +5.6 first difference

−3.4 −1.2 +1 +3.2 second difference

+2.2 +2.2 +2.2 third difference

cubic

Use finite differences to determine the degree of the polynomial that best describes the data.

B

x	−6	−4	−2	0	2	4
y	−30	15	30	34	41	60

+45 +15 +4 +7 +19 first difference

−30 −11 +3 +12 Second difference

+19 +14 +9 third difference

−5 −5 fourth difference

quartic

① STAT 1: Edit
② Enter numbers in L1 and L2
③ STAT → CALC 4: LinReg 5: QuadReg
6: CubicReg 7: QuartReg

Once you have determined the degree of the polynomial that best describes the data, you can use your calculator to create the function.

turn on Diagnostics!

EXAMPLE 2

Using Finite Differences to Write a Function

The table below shows the population of a city from 1950 to 2000. Write a polynomial function for the data. years since 1950

	0	10	20	30	40	50
Year	**1950**	**1960**	**1970**	**1980**	**1990**	**2000**
Population (thousands)	**2853**	**4011**	**5065**	**6720**	**9704**	**14,759**

Lin = 0.8996695
Quad = 0.99025847
Cub = 0.9999994
Quar = 0.99999999 ⟹ best model

$.000077x^4 + 0.1132x^3 - 3.974x^2 + 144.163x + 2852.964$

Often, real-world data can be too irregular for you to use finite differences or find a polynomial function that fits perfectly. In these situations, you can use the regression feature of your graphing calculator. Remember that the closer the R^2-value is to 1, the better the function fits the data.

EXAMPLE 3

Finance Application

The table shows the opening value of a stock index on the first day of trading in various years. Use a polynomial model to estimate the value on the first day of trading in 2002.

years since 1990

Year	Price ($)	Year	Price ($)
4 1994	774	10 2000	4186
5 1995	751	11 2001	2474
6 1996	1053	13 2003	1347
7 1997	1293	14 2004	2011

Lin = 0.256404

Quad = 0.5516815

Cub = 0.627856

Quar = 0.8431598

$$9.2688x^4 - 339.8626x^3 + 4356.7626x^2 - 22616.0956x + 41134.163$$