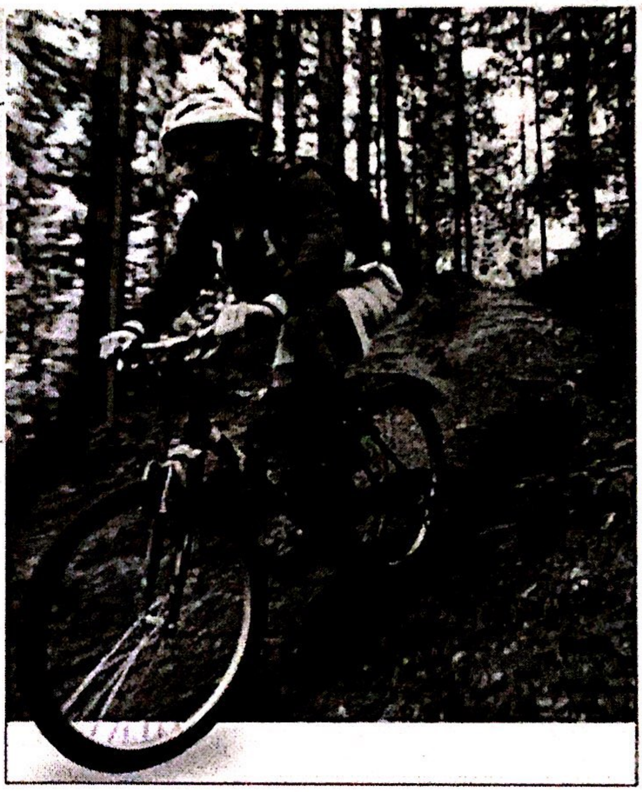
Transforming Polynomial Functions

Why learn this?

Transformations can be used in business to model sales. (See Example 5.)



Lesson Objective(s):

- Transform polynomial functions.

You can perform the same transformations on polynomial functions that you performed on quadratic and linear functions.

$$y = -a(-bx \pm h) \pm K$$

Transformations of *f*(x)

Transformation	*f*(x) Notation	Examples
vertical translation	$f(x)+K$ $f(x)-K$	up down
horizontal translation	$f(x+h)$ $f(x-h)$	left right
vertical stretch/compress	$a \cdot f(x)$	$a>1$ stretch $0<a<1$ (fraction) compress
horizontal stretch/compress	$f(bx)$	$b>1$ compression $0<b<1$ (fraction) stretch
reflection	$-f(x)$ $f(-x)$	across x-axis across y-axis

EXAMPLE 1

Translating a Polynomial Function

For $f(x) = x^3 + 4$, write the rule for each function and sketch its graph.

A $g(x) = f(x) \underbrace{+ 3}_{+k}$ vertical translation up 3

$g(x) = x^3 + 4 + 3$

$g(x) = x^3 + 7$

replace x with x-5

B $g(x) = f(x \underbrace{- 5}_{-h})$ horizontal translation right 5

$g(x) = (x-5)^3 + 4$

EXAMPLE 2

Reflecting Polynomial Functions

Let $f(x) = x^3 - 7x^2 + 6x - 5$. Write a function g that performs each transformation.

A **Reflect $f(x)$ across the x-axis.**

$-f(x)$

$$g(x) = -(x^3 - 7x^2 + 6x - 5)$$
$$g(x) = -x^3 + 7x^2 - 6x + 5$$

B **Reflect $f(x)$ across the y-axis.**

$f(-x)$

$$g(x) = (-x)^3 - 7(-x)^2 + 6(-x) - 5$$
$$g(x) = -x^3 - 7x^2 - 6x - 5$$

EXAMPLE 3

Compressing and Stretching Polynomial Functions

Let $f(x) = x^4 - 4x^2 + 2$. Graph f and g on the same coordinate plane. Describe g as a transformation of f.

A $g(x) = \underbrace{2}f(x)$

$a \cdot f(x)$ $a > 1$

vertical stretch

$g(x) = 2(x^4 - 4x^2 + 2)$

$g(x) = 2x^4 - 8x^2 + 4$

B $g(x) = f(\underbrace{3x})$

$f(bx)$ $b > 1$

horizontal compression

$g(x) = (3x)^4 - 4(3x)^2 + 2$

$g(x) = 81x^4 - 36x^2 + 2$

EXAMPLE 4

Combining Transformations

Write a function that transforms $f(x) = 3x^3 + 6$ in each of the following ways. Support your solution by using a graphing calculator.

A **Stretch vertically by a factor of 2, and shift 3 units left.**

$\underbrace{\text{Stretch vertically by a factor of 2}}_{2 \cdot f(x)}$ and $\underbrace{\text{shift 3 units left}}_{f(x+3)}$

$$2 \cdot f(x+3)$$

$$2(3(x+3)^3 + 6)$$

$$6(x+3)^3 + 12$$

B **Reflect across the x-axis and shift 3 units up.**

$\underbrace{\text{Reflect across the } x\text{-axis}}_{-f(x)}$ and $\underbrace{\text{shift 3 units up}}_{f(x)+3}$

$$-f(x) + 3$$

$$-(3x^3 + 6) + 3$$

$$-3x^3 - 6 + 3$$

$$-3x^3 - 3$$