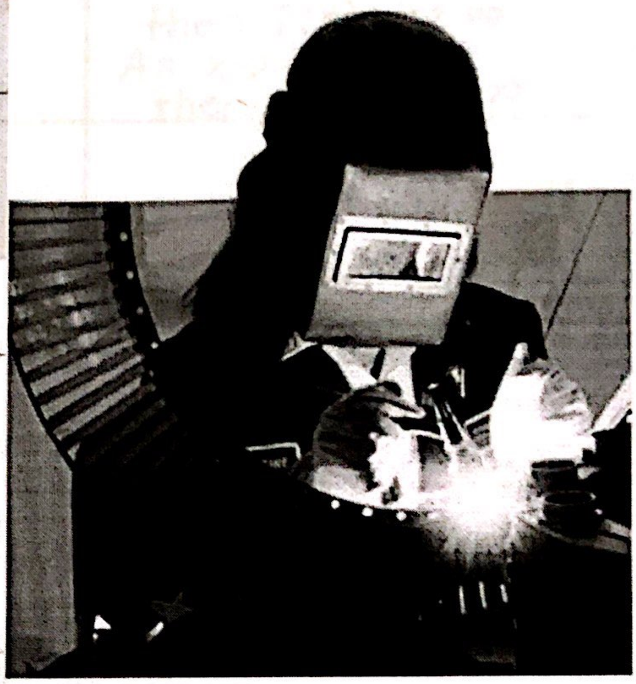
Investigating Graphs of Polynomial Functions



Who uses this?

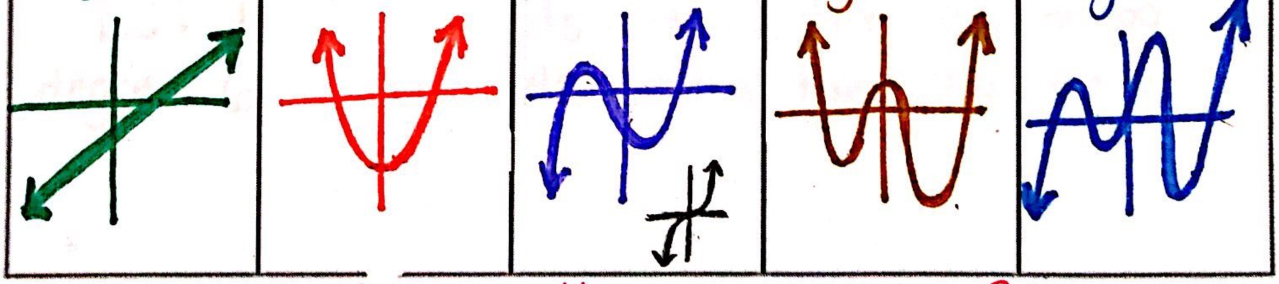
Welders can use graphs of polynomial functions to optimize the use of construction materials. (See Example 5.)

Lesson Objective(s):

- Use properties of end behavior to analyze, describe, and graph polynomial functions.
- Identify and use maxima and minima of polynomial functions to solve problems.

Polynomial functions are classified by their degree. The graphs of polynomial functions are classified by the degree of the polynomial. Each graph, based on the degree, has a distinctive shape and characteristics.

Graphs of Polynomial Functions				
Linear degree = 1	Quadratic degree = 2	Cubic degree = 3	Quartic degree = 4	Quintic degree = 5



Where are the arrows pointing?

End behavior is a description of the values of the function as x approaches positive infinity

The degree and leading coefficient of a polynomial function determine its end behavior. It is helpful when you are graphing a polynomial function to know about the end behavior of the function.

Polynomial End Behavior

P(x) has...	opposite Odd Degree	same Even Degree
Leading coefficient $a > 0$ positive (like the picture)	(left, right) down up As $x \to +\infty$, then $P(x) \to +\infty$ As $x \to -\infty$ then $P(x) \to -\infty$	up up As $x \to +\infty$ then $P(x) \to +\infty$ As $x \to -\infty$ then $P(x) \to +\infty$
Leading coefficient $a < 0$ negative ↳ reflect over the x-axis	up down As $x \to +\infty$ then $P(x) \to -\infty$ As $x \to -\infty$ then $P(x) \to +\infty$	down down As $x \to +\infty$ then $P(x) \to -\infty$ As $x \to -\infty$ then $P(x) \to -\infty$

attached to the highest degree

EXAMPLE 1

Determining End Behavior of Polynomial Functions

Identify the leading coefficient, degree, and end behavior.

A $P(x) = -4x^3 - 3x^2 + 5x + 6$

LC: -4
degree: 3

As $x \to +\infty$, then $P(x) \to -\infty$
As $x \to -\infty$, then $P(x) \to +\infty$

B $R(x) = x^6 - 7x^5 + x^3 - 2$

LC: 1
degree: 6

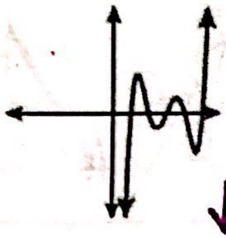
As $x \to +\infty$, then $P(x) \to +\infty$
As $x \to -\infty$, then $P(x) \to +\infty$

EXAMPLE 2

Using Graphs to Analyze Polynomial Functions

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

A

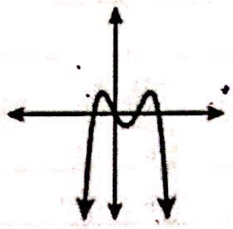


down
up

↓ ↑
opposite = odd

odd degree
positive leading coefficient

B



down
down

↓ ↓
same = even

even degree
negative leading coefficient

Turning point where a graph changes from increasing to decreasing or from decreasing to increasing. A turning point corresponds to a *local maximum* or *minimum*.

Local Maxima and Minima

For a function $f(x)$, $f(a)$ is a local maximum if there is an interval around a such that $f(x) \boxed{<} f(a)$ for every x-value in the interval except a.

For a function $f(x)$, $f(a)$ is a local minimum if there is an interval around a such that $f(x) \boxed{>} f(a)$ for every x-value in the interval except a.

Know it! Note

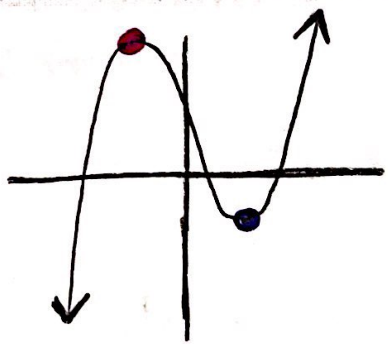
A polynomial function of degree n has at most $n - 1$ turning points and at most n x-intercepts. If the function has n distinct real roots, then it has exactly $n - 1$ turning points and exactly n x-intercepts. You can use a graphing calculator to graph and estimate maximum and minimum values.

a cubic can have at most 2 turning points and 3 x-intercepts

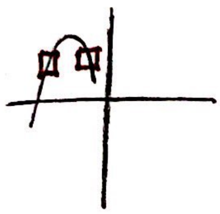
EXAMPLE 4

Determine Maxima and Minima with a Calculator

Graph $g(x) = 2x^3 - 12x + 6$ on a calculator, and estimate the local maxima and minima.



1. put into y=
2. adjust window
3. 2nd TRACE
   - 3: minimum
   - 4: maximum
4. follow directions on the bottom of the screen
   - left bound
   - right bound
   - guess



max: (-1.414, 17.314)

min: (1.414, -5.314)

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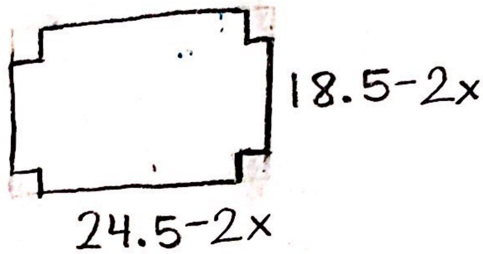
EXAMPLE 5

Industrial Application

A welder plans to construct an open box from an 18.5 ft by 24.5 ft sheet of metal by cutting squares from the corners and folding up the sides. Find the maximum volume of the box and the corresponding dimensions.

$V = lwh$

$V = (24.5 - 2x)(18.5 - 2x)x$



cut out 3.48 ft squares from the corners to have a volume of 704.392 ft^3