

$$1. \sec x - \tan x \sin x = \frac{1}{\sec x}$$

$$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x} = \frac{1}{\sec x}$$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{1}{\sec x}$$

$$\frac{1 - \sin^2 x}{\cos x} = \frac{1}{\sec x}$$

$$\frac{\cos^2 x}{\cos x} = \frac{1}{\sec x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\frac{1}{\sec x} = \frac{1}{\sec x}$$

$$2. \frac{1 + \cos x}{\sin x} = \csc x + \cot x$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \csc x + \cot x$$

$$\csc x + \cot x = \csc x + \cot x$$

$$3. \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$\frac{\frac{1}{\cos \theta} \cdot \sin \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta + \frac{\cos^2 \theta}{\sin \theta}}$$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta \sin \theta}{1} = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta$$

$$4. \frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta} = 1$$

$$\frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = 1$$

$$\frac{1}{\cos\theta} \cdot \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = 1$$

$$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = 1$$

$$\frac{1 - \sin^2\theta}{\cos^2\theta} = 1$$

$$\frac{\cos^2\theta}{\cos^2\theta} = 1$$

$$1 = 1$$

$$5. \cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

$$1 - \sin^2 y - \sin^2 y = 1 - 2\sin^2 y$$

$$1 - 2\sin^2 y = 1 - 2\sin^2 y$$

$$6. \csc^2\theta \tan^2\theta - 1 = \tan^2\theta$$

$$(1 + \cot^2\theta) \tan^2\theta - 1 = \tan^2\theta$$

$$\tan^2\theta + \cot^2\theta \tan^2\theta - 1 = \tan^2\theta$$

$$\tan^2\theta + \frac{1}{\tan^2\theta} \tan^2\theta - 1 = \tan^2\theta$$

$$\tan^2\theta + 1 - 1 = \tan^2\theta$$

$$\tan^2\theta = \tan^2\theta$$

$$7. \frac{\sec^2\theta}{\sec^2\theta - 1} = \csc^2\theta$$

$$\frac{\sec^2\theta}{\tan^2\theta} = \csc^2\theta$$

$$\frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = \csc^2\theta$$

$$\frac{1}{\sin^2\theta} = \csc^2\theta$$

$$\csc^2\theta = \csc^2\theta$$

$$8. \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$\checkmark (\sec^2 x - 1) \sin^2 x = \tan^2 x - \sin^2 x$$

$$\sec^2 x \sin^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\frac{1}{\cos^2 x} \sin^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\tan^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

$$9. (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = 2$$

$$\sin^2 \theta + \cos^2 \theta + \sin^2 \theta + \cos^2 \theta = 2$$

$$1 + 1 = 2$$

$$2 = 2$$

$$10. (\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \csc \theta$$

$$\sin \theta \tan \theta + \sin \theta \cot \theta + \cos \theta \tan \theta + \cos \theta \cot \theta = \sec \theta + \csc \theta$$

$$\sin \theta \left(\frac{\sin \theta}{\cos \theta} \right) + \sin \theta \left(\frac{\cos \theta}{\sin \theta} \right) + \cos \theta \left(\frac{\sin \theta}{\cos \theta} \right) + \cos \theta \left(\frac{\cos \theta}{\sin \theta} \right) = \sec \theta + \csc \theta$$

$$\frac{\sin^2 \theta}{\cos \theta} + \cos \theta + \sin \theta + \frac{\cos^2 \theta}{\sin \theta} = \sec \theta + \csc \theta$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} = \sec \theta + \csc \theta$$

$$\frac{1}{\cos \theta} + \frac{1}{\sin \theta} = \sec \theta + \csc \theta$$

$$\sec \theta + \csc \theta = \sec \theta + \csc \theta$$

★

$$11. \frac{\tan \theta - 1}{\tan \theta + 1} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\frac{\frac{\sin \theta}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta} + 1} = \frac{1 - \frac{\cos \theta}{\sin \theta}}{1 + \frac{\cos \theta}{\sin \theta}}$$

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \cdot \frac{\cos \theta}{\sin \theta + \cos \theta} = \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \cdot \frac{\sin \theta}{\sin \theta + \cos \theta}$$

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

★ manipulate both sides

$$12. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\frac{1 - \tan^2 x}{\sec^2 x} = 1 - 2\sin^2 x$$

$$\frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x} = 1 - 2\sin^2 x$$

$$\cos^2 x - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = 1 - 2\sin^2 x$$

$$\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$(1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$$

$$1 - 2\sin^2 x = 1 - 2\sin^2 x$$

$$13. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{\sin x (\sin^2 x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{\sin x (1 - \cos^2 x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{\sin x (1 + \cos x) (1 - \cos x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{1}{\sin x (1 - \cos x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{1}{\sin x} \cdot \frac{1}{1 - \cos x} = \frac{\csc x}{1 - \cos x}$$

$$\csc x \cdot \frac{1}{1 - \cos x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1 - \cos x} = \frac{\csc x}{1 - \cos x}$$

$$14. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$\therefore (\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x) = \csc^2 x + \cot^2 x$$

$$\csc^2 x + \cot^2 x = \csc^2 x + \cot^2 x$$

$$15. \frac{\tan \theta}{\sec \theta} + \frac{\cot \theta}{\csc \theta} = \sin \theta + \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{1} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1} = \sin \theta + \cos \theta$$

$$\sin \theta + \cos \theta = \sin \theta + \cos \theta$$

$$16. \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

$$\frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} = \sin y$$

$$\frac{\frac{\sin y \cos y + \sin y}{\cos y}}{\frac{\cos y + 1}{\cos y}} = \sin y$$

$$\frac{\sin y \cos y + \sin y}{\cos y} \cdot \frac{\cos y}{1 + \cos y} = \sin y$$

$$\frac{\sin y (\cos y + 1)}{1 + \cos y} = \sin y$$

$$\sin y = \sin y$$