Factoring Polynomials

Factor: $x^2-7x+12$

$(x-4)(x-3)$

multiply to c
add to b

12
1 12
2 6
-3 -4

Who uses this?

Ecologists may use factoring polynomials to determine when species might become extinct. (See Example 4.)

Lesson Objective(s):

- Use the Factor Theorem to determine factors of a polynomial.
- Factor the sum and difference of two cubes.

Recall that if a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.

The Remainder Theorem states that if a polynomial is divided by $(x - a)$, the remainder is the value of the function at a. So, if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factor Theorem

THEOREM	EXAMPLE
$(x-a)$ is a factor if the remainder is $\boxed{\text{ZERO}}$	

1. use long or synthetic division to find the remainder
2. remainder $= 0$, YES $(x-a)$ is a factor
   remainder $\neq 0$, NO $(x-a)$ is not a factor

EXAMPLE 1

Determining Whether a Linear Binomial is a Factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A $(x-3);\ P(x)=x^2+2x-3$

$(x\quad)(x\quad)$

$$\begin{array}{r|rrr} 3 & 1 & 2 & -3 \\ & \downarrow & 3 & 15 \\ \hline & 1 & 5 & 12 \end{array}$$

needs to be zero

no; $(x-3)$ is not a factor of $P(x)$

B $(x+4);\ P(x)=2x^4+8x^3+2x+8$

$(\quad)(\quad)(\quad)(\quad)(\quad)$

gap

$$\begin{array}{r|rrrrr} -4 & 2 & 8 & 0 & 2 & 8 \\ & \downarrow & -8 & 0 & 0 & -8 \\ \hline & 2 & 0 & 0 & 2 & 0 \end{array}$$

yes; $(x+4)$ is a factor of $P(x)$

$\boxed{C}$ $(x-3);\ P(x)=4x^6-12x^5+2x^3-6x^2-5x+10$

gap

$$\begin{array}{r|rrrrrrr} 3 & 4 & -12 & 0 & 2 & -6 & -5 & 10 \\ & \downarrow & 12 & 0 & 0 & 6 & 0 & -15 \\ \hline & 4 & 0 & 0 & 2 & 0 & -5 & -5 \end{array}$$

no; $(x-3)$ is not a factor of $P(x)$

You are already familiar with methods for factoring quadratic expressions. You can factor polynomials of higher degrees using many of the same methods you previously learned.

EXAMPLE 2

Factoring by Grouping

Factor $x^3 + 3x^2 - 4x - 12$.

① Group terms by twos
② Factor out GCF

$x^2(x+3) - 4(x+3)$

the stuff inside ( ) must match

③ Combine terms on the outside + inside

$(x^2-4)(x+3)$

Difference of squares

$(x+2)(x-2)(x+3)$

$a^2 - b^2 = (a+b)(a-b)$

$\boxed{B}$ $3x^3 + x^2 - 27x - 9$

$x^2(3x+1) - 9(3x+1)$

$(x^2-9)(3x+1)$

$(x+3)(x-3)(3x+1)$

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$(x^2-9)(3x+1)$

$\downarrow$

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Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes

METHOD	ALGEBRA
Sum of two cubes	$a^3+b^3=(a+b)(a^2-ab+b^2)$
Difference of two cubes	$a^3-b^3=(a-b)(a^2+ab+b^2)$

EXAMPLE 3

Factoring the Sum or Difference of Two Cubes

Factor each expression.

A $5x^4+40x$ GCF: $5x$

$5x(x^3+8)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$\sqrt[3]{x^3}=x=a$

$\sqrt[3]{8}=2=b$

$5x(x+2)(x^2-2x+4)$

B $8y^3-27$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$\sqrt[3]{8y^3}=2y=a$

$\sqrt[3]{27}=3=b$

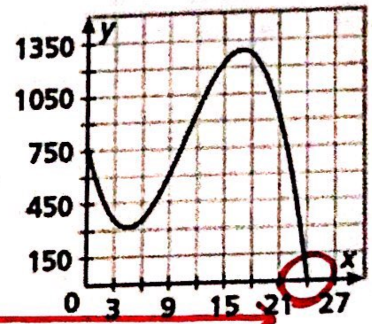
$(2y-3)(4y^2+6y+9)$

You can also use a graph to help you factor a polynomial. Recall that the real zeros of a function appear as x-intercepts on its graph. By the Factor Theorem, if you can determine the zeros of a polynomial function from its graph, you can determine the corresponding factors of the polynomial.

EXAMPLE 4

Ecology Application

The population of an endangered species of bird in the years since 1990 can be modeled by the function $P(x) = -x^3 + 32x^2 - 224x + 768$. Identify the year that the bird will become extinct if the model is accurate and no protective measures are taken. Use the graph to factor $P(x)$.



factor: $(x-24)$

24	−1	32	−224	768
	↓	−24	192	−768
	−1	8	−32	0