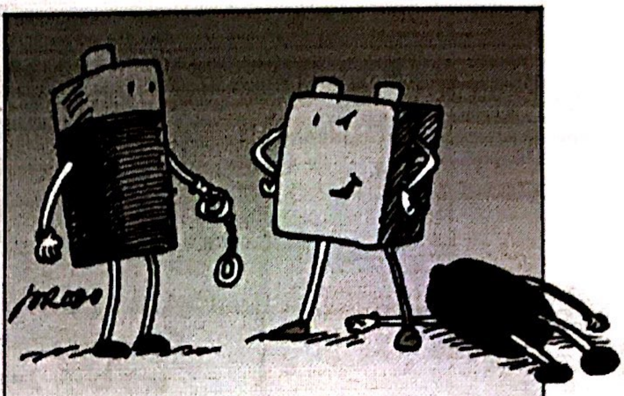
Dividing Polynomials

Who uses this?

Electricians can divide polynomials in order to find the voltage in an electrical system.(See Example 4.)



"Okay, Copper—what's the charge? Assault and Battery? I have contacts, you know"

Lesson Objective(s):

- Use long division and synthetic division to divide polynomials.

Polynomial long division is a method for dividing a polynomial by another polynomial of a lower degree. It is very similar to dividing numbers.

Arithmetic Long Division

$$\begin{array}{r} 23 \\ 12\,\overline{)\,277} \\ -24 \\ \hline 37 \\ -36 \\ \hline 1 \end{array}$$

$23\frac{1}{12}$

Polynomial Long Division

$$\begin{array}{r} 2x+11 \\ x-2\,\overline{)\,2x^2+7x+7} \\ -\cancel{2x^2}+4x \\ \hline \cancel{11x}+7 \\ -\cancel{11x}+22 \\ \hline 29 \end{array}$$

$2x+11+\frac{29}{x-2}$

EXAMPLE 1

Using Long Division to Divide Polynomials

Divide by using long division.

$(4x^2+3x^3+10) \div (x-2)$

① standard form

$3x^3+4x^2+10$

② any gaps?

$$\begin{array}{r} 3x^2+10x+20 \\ x-2\,\overline{)\,3x^3+4x^2+0x+10} \\ -\cancel{3x^3}+6x^2 \\ \hline \cancel{10x^2}+0x \\ -\cancel{10x^2}+20x \\ \hline \cancel{20x}+10 \\ -\cancel{20x}+40 \\ \hline 50 \end{array}$$

$3x^2+10x+20+\frac{50}{x-2}$

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shorthand method of dividing a polynomial by a linear binomial by using only the coefficients. For synthetic division to work, the polynomial must be written in standard form, using 0 as a coefficient for any missing terms, and the divisor must be in the form *(x - a)*.

Synthetic Division Method

Divide $(2x^2 + 7x + 9) \div (x + 2)$ by using synthetic division.

WORDS	NUMBERS
Step 1 Write the coefficients of the dividend, 2, 7, and 9. In the upper left corner, write the value of *a* for the divisor $(x - a)$. So $a = -2$. Copy the first coefficient in the dividend below the horizontal bar.	
Step 2 Multiply the first coefficient by the divisor, and write the product under the next coefficient. Add the numbers in the new column.	
Repeat Step 2 until additions have been completed in all columns. Draw a box around the last sum.	
Step 3 The quotient is represented by the numbers below the horizontal bar. The boxed number is the remainder. The others are the coefficients of the polynomial quotient, in order of decreasing degree.	

EXAMPLE 2

Using Synthetic Division to Divide by a Linear Binomial

Divide by using synthetic division. $(2x^2 + 7x + 9) \div (x + 2)$

opposite

$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ & \downarrow & -4 & -6 \\ \hline & 2 & 3 & \boxed{3} \end{array}$$

multiply

$$2x + 3 + \frac{3}{x+2}$$

Divide by using synthetic division.

B $(x^4 - 2x^3 + 3x + 1) \div (x - 3)$

gap!

opposite!

$$
\begin{array}{r|rrrrr}
3 & 1 & -2 & 0 & 3 & 1 \\
  & \downarrow & 3 & 3 & 9 & 36 \\
\hline
  & 1 & 1 & 3 & 12 & \big| \, 37
\end{array}
$$

multiply

$$1x^3 + 1x^2 + 3x + 12 + \frac{37}{x-3}$$

You can use synthetic division to evaluate polynomials. This process is called synthetic substitution. The process of synthetic substitution is exactly the same as the process of synthetic division, but the final answer is interpreted differently, as described by the Remainder Theorem.

Remainder Theorem

THEOREM	EXAMPLE

EXAMPLE 3

Using Synthetic Substitution

Use synthetic substitution to evaluate the polynomial for the given value.

A $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$ $(x^3 - 4x^2 + 3x - 5) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 3 & -5 \\ & \downarrow & 4 & 0 & 12 \\ \hline & 1 & 0 & 3 & 7 \end{array}$$

$P(4) = 7$

Use synthetic substitution to evaluate the polynomial for the given value.

B $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -\frac{1}{2}$

gap!

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 4 & 2 & 0 & 3 & 5 \\ & \downarrow & -2 & 0 & 0 & -\frac{3}{2} \\ \hline & 4 & 0 & 0 & 3 & \frac{7}{2} \end{array}$$

$P(-\frac{1}{2}) = \frac{7}{2}$