Polynomials

Who uses this?

Doctors can use polynomials to model blood flow. (See Example 4.)

Lesson Objective(s):

- Identify, evaluate, add, and subtract polynomials.
- Classify and graph polynomials.

(1) **Monomial** a number or a product of numbers and variables with whole number exponents. **Polynomial** a monomial or a sum or difference of monomials. Each monomial in a polynomial is a term. Because a monomial has only one term, it is the simplest type of polynomial.

(2) Binomial

Trinomial

(3)

4 or more

Polynomials have no variables in denominators or exponents, no roots or absolute values of variables, and all variables have whole number exponents.

	monomial	binomial	monomial	monomial	binomial
Polynomials:	$3x^4$	$2z^{12} + 9z^3$	$\frac{1}{2}a^7$	$0.15x^{101}$	$3t^2 - t^3$
Not polynomials:	3^x	$2b^3 - 6b$	$\frac{8}{5\sqrt{2}}$	$\frac{1}{2}\sqrt{x}$	$m^{0.75} - m$

Degree of a monomial — the sum of the exponents of the variables.

add

EXAMPLE 1

Identifying the Degree of a Monomial

Identify the degree of each monomial.

A x^4 — 4^{th} quartic

B 12 — constant

C $4a^2b^1$ — 3^{rd} cubic

D $x^3y^4z^1$ — 8^{th} degree

Degree of a polynomial is given by the term with the greatest degree, ignore the rest. A polynomial with one variable is in standard form when its terms are written in descending order by degree. So, in standard form, the degree of the first term indicates the degree of the polynomial, leading coefficient the coefficient of the first term.

Standard Form

Leading coefficient — Degree of polynomial

$5x^3 + 8x^2 + 3x^1 - 17^0$

Degree of term: 3 2 1 0

A polynomial can be classified by its number of terms. A polynomial with two terms is called binomial and a polynomial with three terms is called trinomial. A polynomial can also be classified by its degree.

Know it! Note

Classifying Polynomials by Degree		
Name	**Degree**	**Example**
Constant $y = c$	0	3
Linear $y = x^1$	1	$x + 2$
Quadratic $y = x^2$	2	$x^2 - 4x + 1$
Cubic $y = x^3$	3	$x^3 + 2x^2 - 5x + 7$
Quartic $y = x^4$	4	$5x^4 - x^3 - x^2 + 2x - 1$
Quintic $y = x^5$	5	$6x^5 + 3x^4 - 8x^3 + 2x^2 - x + 273$
n^{th} degree	6, 7, 8, ...	

EXAMPLE 2

Classifying Polynomials

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

A $2x + 4x^3 - 1$

$4x^3 + 2x - 1$

LC: 4

Deg: 3

Terms: 3

Cubic Trinomial

B $7x^3 - 11x + x^5 - 2$

$0x^5 + 7x^3 - 11x - 2$

LC: 1

Deg: 5

Terms: 4

Quintic Polynomial

To add or subtract polynomials, combine like terms. You can add or subtract horizontally or vertically.

EXAMPLE 3

Adding and Subtracting Polynomials

Add or subtract. Write your answer in standard form.

A $(3x^2 + 7 + x) + (14x^3 + 2 + x^2 - x)$ parenthesis do not matter

$3x^2 + 7 + x + 14x^3 + 2 + x^2 - x$

$14x^3 + 4x^2 + 9$

$$\begin{array}{r} 3x^2 + 7 + x \\ 14x^3 + x^2 + 2 - x \\ \hline 14x^3 + 4x^2 + 9 \end{array}$$

Add or subtract. Write your answer in standard form.

B $(1 - x^2) - (3x^2 + 2x - 5)$ parenthesis matter! must distribute!

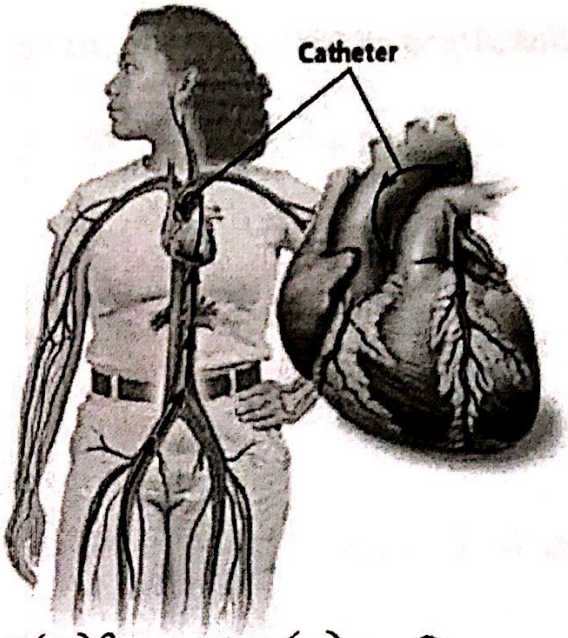
$1 - x^2 - 3x^2 - 2x + 5$

$-4x^2 - 2x + 6$

a function whose rule is a polynomial. In this course, you will study only polynomial functions with one variable.

EXAMPLE 4

Medical Application



Cardiac output is the amount of blood pumped through the heart. The output is measured by a technique called dye dilution. A doctor injects dye into a vein near the heart and measures the amount of dye in the arteries over time.

The cardiac output of a particular patient can be approximated by the function $f(t) = 0.0056t^3 - 0.22t^2 + 2.33t$, where t represents time (in seconds after injection, $0 \le t \le 23$) and $f(t)$ represents the concentration of dye (in milligrams per liter).

a. Evaluate $f(t)$ for $t = 0$ and $t = 3$.

$f(0) = 0.0056(0)^3 - 0.22(0)^2 + 2.33(0) = 0$

$f(3) = 0.0056(3)^3 - 0.22(3)^2 + 2.33(3) = 5.1612$

b. Describe what the values of the function from part a represent.

At 0 seconds after injection, there is no dye in your system yet.

At 3 seconds after injection, there is 5.1612 mg per liter of dye in your system.

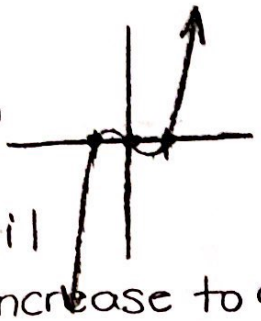
Graphing polynomial functions can be a challenge. Throughout this chapter, you will learn skills for analyzing, describing, and graphing higher-degree polynomials. Until then, the graphing calculator will be a useful tool.

EXAMPLE 5

Graphing Higher-Degree Polynomials on a Calculator

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros. crosses the x-axis

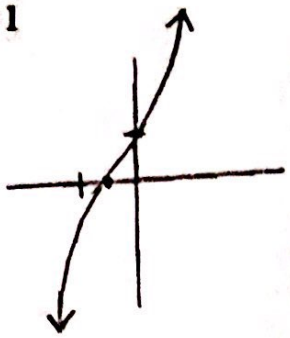
A $f(x) = x^3 - x$



increase from $-\infty$ until about $-1/2$, then decrease until about $1/2$, then increase to ∞

3 real zeros

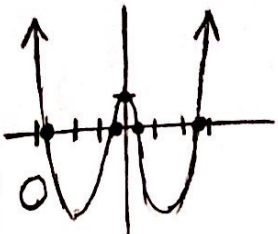
B $f(x) = 3x^3 + 2x + 1$



increase from left to right

1 real zero

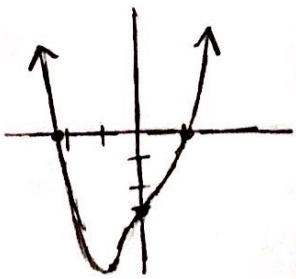
C $h(x) = x^4 - 8x^2 + 1$



decreasing $-\infty$ until about -1.8, then increasing until 0 then decreasing until about 1.8 and then increasing to ∞

4 real zeros

D $k(x) = x^4 + x^3 - x^2 + 2x - 3$



decrease from $-\infty$ until about -1 then increase to ∞

2 real zeros