Transforming <u>Linear</u> Functions

$$y = x$$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$
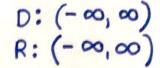
Why learn this?

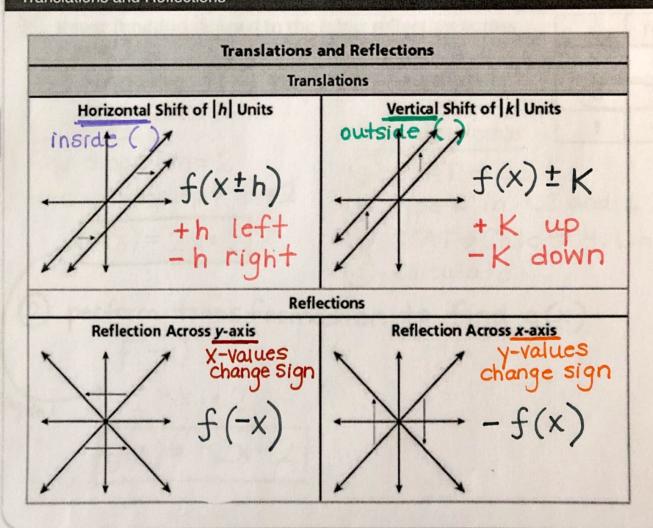Transformations allow you to visualize and compare many different functions at once.

Lesson Objective(s):

- Transform linear functions.
- Solve problems involving linear transformations.

You have learned to transform functions by transforming each point. Transformations can also be expressed by using function notation.

Translations and Reflections

Translations and Reflections	
Translations	
Horizontal Shift of $|h|$ Units	**Vertical Shift of $|k|$ Units**
inside ( )	outside ( )
$f(x \pm h)$	$f(x) \pm k$
+h left	+K up
−h right	−K down
Reflections	
Reflection Across y-axis	**Reflection Across x-axis**
x-values change sign	y-values change sign
$f(-x)$	$-f(x)$

EXAMPLE 1

Translating and Reflecting Linear Functions

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

A $f(x) = 2x + 3$; vertical translation 4 units up

$f(x) \pm K$ $f(x) + 4$

plug in! $(2x + \underline{3}) + \underline{4}$

$$\boxed{g(x) = 2x + 7}$$

horizontal translation
5 units left

$f(x \pm h)$
$f(x + 5)$

$2(x + 5) + 3$
$2x + 10 + 3$

$$\boxed{g(x) = 2x + 13}$$

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

B linear function defined in the table; reflection across
y-axis

x	f(x)
−1	0
0	2
1	4

(+2, +2)

① find the $f(x)$ function $y = mx + b$

by hand
ⓐ find slope $m = 2$
ⓑ find y-intercept $b = 2$

$$\boxed{f(x) = 2x + 2}$$

in calculator
ⓐ STAT → Edit
ⓑ type # in L1 and L2
ⓒ STAT → Calc → 4: LinReg
ⓓ Calculate
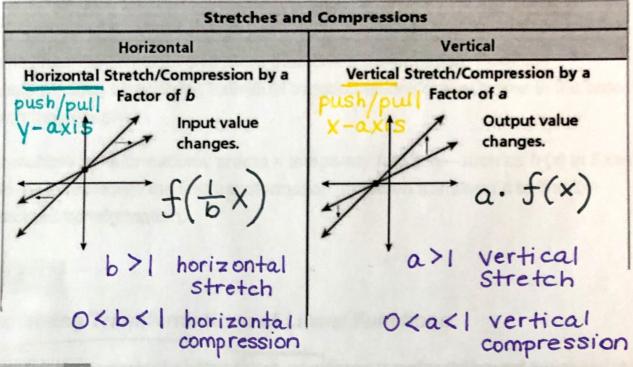
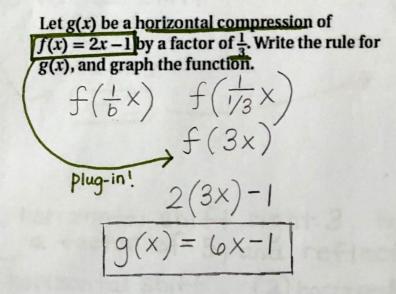② perform transformation to find $g(x)$
$f(-x)$

plug in! $2(-x) + 2$

$$\boxed{g(x) = -2x + 2}$$

Stretches and compressions change the slope of a linear function. If the line becomes steeper, the function has been stretched vertically or compressed horizontally. If the line becomes flatter, the function has been compressed vertically or stretched horizontally.

Stretches and Compressions	
Horizontal	**Vertical**
Horizontal Stretch/Compression by a Factor of b	**Vertical Stretch/Compression by a Factor of a**
push/pull *y-axis* Input value changes. $f\left(\frac{1}{b}x\right)$	*push/pull* *x-axis* Output value changes. $a \cdot f(x)$
$b > 1$ horizontal stretch $0 < b < 1$ horizontal compression	$a > 1$ vertical stretch $0 < a < 1$ vertical compression

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EXAMPLE 2

Stretching and Compressing Linear Functions

Let g(x) be a horizontal compression of $f(x) = 2x - 1$ by a factor of $\frac{1}{3}$. Write the rule for g(x), and graph the function.

$f\left(\frac{1}{b}x\right) \quad f\left(\frac{1}{1/3}x\right)$

$f(3x)$

plug-in!

$2(3x) - 1$

$\boxed{g(x) = 6x - 1}$

vertical stretch by a factor of 4

$a \cdot f(x) \qquad 4 \cdot f(x)$

$4(2x-1)$

$\boxed{g(x) = 8x - 4}$

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Some linear functions involve more than one transformation. Combine transformations by applying individual transformations one at a time in the order in which they are given.

For multiple transformations, create a temporary function—such as $h(x)$ in Example 3 below—to represent the first transformation, and then transform it to find the combined transformation.

EXAMPLE 3

Combining Transformations of Linear Functions

Let $g(x)$ be a vertical shift of $f(x) = x$ down 2 units followed by a vertical stretch by a factor of 5. Write the rule for $g(x)$.

① vertical shift
$$f(x) \pm K$$
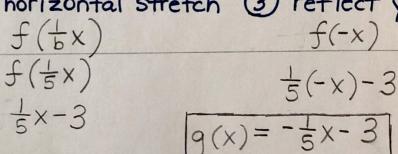$$f(x) - 2$$
$$\boxed{x - 2}$$

② vertical stretch
$$a \cdot f(x)$$
$$5 \cdot f(x)$$
$$5(x-2)$$
$$\boxed{g(x) = 5x - 10}$$

horizontal shift right 3, horizontal stretch by a factor of 5, and reflection over y-axis

① horizontal shift
$$f(x \pm h)$$
$$f(x-3)$$
$$x-3$$

② horizontal stretch
$$f(\tfrac{1}{b}x)$$
$$f(\tfrac{1}{5}x)$$
$$\tfrac{1}{5}x - 3$$

③ reflect y-axis
$$f(-x)$$
$$\tfrac{1}{5}(-x) - 3$$
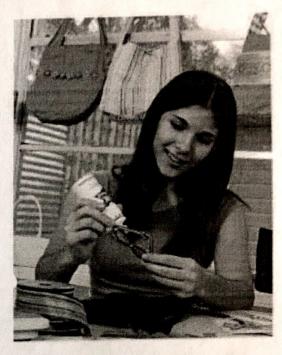$$\boxed{g(x) = -\tfrac{1}{5}x - 3}$$

EXAMPLE 4

Fund-raising Application

The Dance Club is selling beaded purses as a fund-raiser. The function $R(n) = 12.5n$ represents the club's <u>revenue</u> in dollars where n is the number of purses sold.

↓ selling
$12.50 per purse

a. The club paid $75 for the materials to make the purses. Write a new function, P(n), for the club's profit.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$
$$P(n) = 12.5n - 75$$

b. Describe the transformation from R(n) to P(n).

vertical translation down 75

LESSON 1-3 Problem Solving

Transforming Linear Functions

The students in Ms. Hari's English class are planning to print a
booklet of their creative writings. Use the table of publishing prices.

1. The students decide to print a booklet containing
black and white text only. Write a function, $C(p)$, to
show the cost of printing a booklet of p pages with
a cover that also has text only.

$$C(p) = 0.55p + 2.25$$

2. Julie wants the booklet cover to have a color
graphic. Write a new function, $J(p)$, to show this
cost for a booklet of p pages.

$$J(p) = 0.55p + 3.50$$

Publishing Prices		
	Text Only	Color Graphic
Per page	$0.55	$1.25
Cover	$2.25	$3.50

3. What is the slope of each function? What does the slope tell you
about the relationship of the lines?

$0.55   same slope = parallel lines

4. What is the y-intercept of each function? What is represented by
the y-intercept?

$2.25, $3.50   cost of the cover

5. Describe the transformation that has been applied to the graph by
the decision to change the cover.

Vertical translation up $1.25

6. Oscar suggests that the booklet have 30 pages, one for each person in
the class. What is the cost of printing 50 booklets using the function $J(p)$?

$$J(p) = 0.55(30) + 3.50 = \$20 \text{ per booklet}$$
$$\times 50 = \$1000 \text{ total}$$

Choose the letter for the best answer.

7. Lee writes a function for the cost of p
pages, all in color, with a plain text
cover. What transformation does this
apply to the graph of $C(p)$?

   A Horizontal stretch

   B Horizontal compression

   C Vertical stretch

   D Vertical compression

8. Tina finds a printer who will print text
pages at $0.25 a page, with a color
cover for $2.00. Using this printer, what
is the cost of 50 booklets of 30 pages
each?

   A $950

   B $725

   C $600

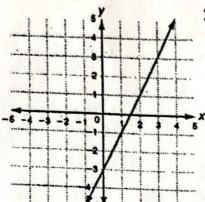   D $475

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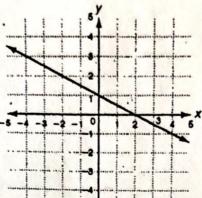**LESSON 1-3**

Practice

Transforming Linear Functions

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

1.

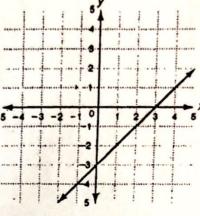horizontal translation
left 3 units

_____

2.

vertical compression by
a factor of $\dfrac{1}{5}$

_____

3.

reflection across the
y-axis

_____

4. linear function defined by the table; horizontal stretch by
a factor of 2.3

_____

x	−5	0	7
y	−3	7	21

5. $f(x) = 1.7x - 3$; vertical compression by a factor of 0.7

_____

Let $g(x)$ be the indicated combined transformation of $f(x) = x$. Write
the rule for $g(x)$.

6. vertical translation down 2 units followed by a

   horizontal compression by a factor of $\dfrac{2}{5}$

   _____

7. horizontal stretch by a factor of 3.2 followed by
   a horizontal translation right 3 units

   _____

Solve.

8. The Red Cab Taxi Service used to charge $1.00 for the first $\dfrac{1}{5}$ mile and $0.75 for each

   additional $\dfrac{1}{5}$ mile. The company just raised its rates by a factor of 1.5.

   a. Write a new price function $g(x)$ for a taxi ride.

   _____

   b. Describe the transformation(s) that have been applied.

   _____