

Precalculus - Unit 9 Day 2

Limits from Functions

Finding Limits Analytically

We have already seen that, in some cases, limits can be found by directly substituting a value into a function, like the examples below.

can I plug in the # and find the answer?

Ex 9:

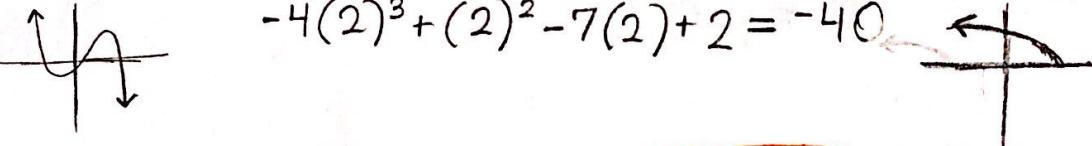
$$(a) \lim_{x \rightarrow 4} 3x^2 - 2 = 3(-4)^2 - 2 = 46$$

$$(b) \lim_{x \rightarrow 0} (-8) = -8$$

$$(c) \lim_{x \rightarrow 2} -4x^3 + x^2 - 7x + 2 =$$

$$-4(2)^3 + (2)^2 - 7(2) + 2 = -40$$

$$(d) \lim_{x \rightarrow -1} \sqrt{5-2x} = -\sqrt{7}$$



But what happens when direct substitution **DOES NOT WORK!**!!!!?

get a zero in the denominator

Ex 10:

$$(a) \lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \frac{(x+4)(x-4)}{x-4} = x+4 = 8$$

① Try to factor

$$(b) \lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \frac{(x^2 + 1)(x^2 - 1)}{x - 1} = \frac{(x^2 + 1)(x + 1)(x - 1)}{(x^2 + 1)(x + 1)}$$

② Try to multiply by the conjugate

$$(c) \lim_{x \rightarrow 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \frac{(x-9)(\sqrt{x}+3)}{x+3\sqrt{x}-3\sqrt{x}-9} = \frac{(x-9)(\sqrt{x}+3)}{x-9} = \sqrt{x}+3 = \sqrt{9}+3=6$$

$$(d) \lim_{x \rightarrow 25} \frac{\sqrt{x}+5}{x-25} \frac{(\sqrt{x}-5)}{(\sqrt{x}-5)} = \frac{(\sqrt{x}-5)}{(\sqrt{x}-5)} = \boxed{DNE}$$

*it may not work
and, in that case,
check #'s around it

Limits at Infinity

VA: set denominator
= 0

HA: top heavy none
bottom heavy $y = 0$
equal $y = \frac{a}{b}$

Ex 11:

For the following functions, find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. Then find any vertical or horizontal asymptotes and any holes.

$$(a) f(x) = 2x^2 - 3x + 8$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$(b) f(x) = \frac{5x+20}{x^2+9x+20} = \frac{5}{x+5}$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

hole @ $x = -4$

VA @ $x = -5$

HA @ $y = 0$

$$\lim_{x \rightarrow \infty} f(x)$$

$$\lim_{x \rightarrow -\infty} f(x)$$

VA @ $x = -\frac{9}{2}$ or -4.5

HA @ $y = -\frac{3}{2}$ or -1.5

Ex 12 (Mixed Review):

$$\lim_{x \rightarrow 2} (3x^3 - 7) = \\ 3(2)^3 - 7 = 17$$

$$\lim_{x \rightarrow 0} \frac{9x^5 - 4x^3}{7x^6 - 3x^3} = \frac{x^3(9x^2 - 4)}{x^3(7x^3 - 3)}$$

$$\frac{9x^2 - 4}{7x^3 - 3} = \frac{9(0)^2 - 4}{7(0)^3 - 3} = \frac{4}{3}$$

$$\lim_{x \rightarrow \infty} \frac{-3x^5 + 21x - 120}{14x^2 - 11x + 354} = -\infty$$

Ex 13 (Mixed Review):

Using the graph on the right, find the indicated limits.

$$a. \lim_{x \rightarrow -2^-} f(x) = -\infty$$

$$b. \lim_{x \rightarrow 5^+} f(x) = \infty$$

$$c. \lim_{x \rightarrow \infty} f(x) = -1$$

$$d. \lim_{x \rightarrow 5^-} f(x) = -\infty$$

