

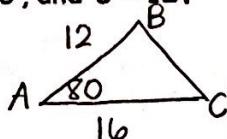
Name: \_\_\_\_\_ Class Period: \_\_\_\_\_

## Precalculus - Unit 3 Day 2 Law of Cosines

For an oblique triangle given **SAS** or **SSS**, we do not have adequate information to use the Law of Sines ...

**Example 1:**

Solve  $\triangle ABC$  where  $A = 80^\circ$ ,  $b = 16$ , and  $c = 12$ .



$$a^2 = 12^2 + 16^2 - 2(12)(16)\cos 80^\circ$$

$$a = 18.26$$

\*Find smallest  $\angle$  first!

$$\frac{\sin C}{12} = \frac{\sin 80}{18.26}$$

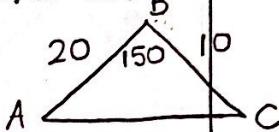
$$C = \sin^{-1}\left(\frac{12\sin 80}{18.26}\right)$$

$$\angle C = 40.33^\circ \quad \angle B = 59.67^\circ$$

**Example 2:** Solve  $\triangle ABC$  where  $B = 150^\circ$ ,  $a = 10$ ,  $c = 20$ .

**Example 2:**

Solve  $\triangle ABC$  where  $B = 150^\circ$ ,  $a = 10$ ,  $c = 20$ .



$$b^2 = 10^2 + 20^2 - 2(10)(20)\cos 150^\circ$$

$$b = 29.09$$

$$\frac{\sin A}{10} = \frac{\sin 150}{29.09}$$

$$A = \sin^{-1}\left(\frac{10\sin 150}{29.09}\right)$$

$$\angle A = 9.90^\circ \quad \angle B = 20.10^\circ$$

**Example 3:** Solve  $\triangle ABC$  where  $A = 25^\circ$ ,  $b = 9$ , and  $c = 12$ .

\*Note: In this situation when you are given an acute angle with **SAS**, there is a possibility that one of the other angles is obtuse. The Inverse Sine function will only produce an acute angle, because the restricted range is  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  or  $[-90^\circ, 90^\circ]$ .

So you can never use the Law of Sines to solve for an obtuse angle!

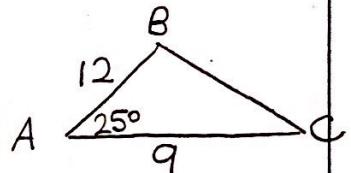
$$a^2 = 9^2 + 12^2 - 2(9)(12)\cos 25^\circ$$

$$a = 5.41$$

$$\frac{\sin B}{9} = \frac{\sin 25}{5.41}$$

$$B = \sin^{-1}\left(\frac{9\sin 25}{5.41}\right)$$

$$\angle B = 44.67^\circ$$



$$\angle C = 110.33^\circ$$

- NOTE -** In the case of solving a triangle given SSS, always find the largest angle first!
- > The inverse of cosine has a restricted range of  $[0, \pi]$  or  $[0^\circ, 180^\circ]$  whereas the inverse of sine only has a restricted range of  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  or  $[-90^\circ, 90^\circ]$ .
  - > In other words, the Inverse Cosine is the only function to produce an obtuse angle; therefore you need to find the largest angle first.

**Example 4:** Solve  $\triangle ABC$  where  $a = 6$ ,  $b = 8$ , and  $c = 12$ .

$$12^2 = 6^2 + 8^2 - 2(6)(8)\cos C$$

$$144 = 36 + 64 - 96\cos C$$

$$-36 \quad \swarrow -36 \quad -64$$

$$-64$$

$$\frac{44}{-96} = \frac{-96\cos C}{-96}$$

$$-\frac{11}{24} = \cos C$$

$$C = \cos^{-1}\left(-\frac{11}{24}\right)$$

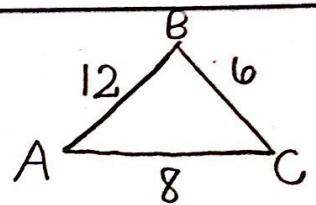
$$\angle C = 117.28^\circ$$

$$\frac{\sin A}{6} = \frac{\sin 117.28}{12}$$

$$A = \sin^{-1}\left(\frac{6\sin 117.28}{12}\right)$$

$$\angle A = 26.38^\circ$$

$$\angle B = 36.34^\circ$$



**Example 5:** Solve  $\triangle ABC$  where  $a = 7$ ,  $b = 15$ , and  $c = 19$ .

$$19^2 = 7^2 + 15^2 - 2(7)(15)\cos C$$

$$361 = 49 + 225 - 210\cos C$$

$$-49 \quad \swarrow -49 \quad -225$$

$$-225$$

$$87 = -210\cos C$$

$$-\frac{29}{70} = \cos C$$

$$C = \cos^{-1}\left(-\frac{29}{70}\right)$$

$$\angle C = 114.47^\circ$$

$$\frac{\sin A}{7} = \frac{\sin 114.47}{19}$$

$$A = \sin^{-1}\left(\frac{7\sin 114.47}{19}\right)$$

$$\angle A = 19.59^\circ$$

$$\angle B = 45.94^\circ$$

