

Precalculus - Unit 2 Day 5 Solving Trig Equations

Identities are true for all values of x and we use them to simplify other trig identities.

Equations are true only for specific values of x . When we solve an equation, we are finding those specific values of x that satisfy the equation.

- The primary goal in solving trigonometric equations is to isolate the trig function involved in the equation. Then, consider the values for "x" that make that equation true.
- If there is no restriction on domain, then state all the possible values by determining the first solution and then the interval by which all other solutions will occur. Use "n" to represent any integer.

Solve over the interval $[0, 2\pi)$.

$$2x - 1 = 0$$

<p>1. $2\sin x - 1 = 0$ $\quad +1 +1$ $\frac{2\sin x}{2} = \frac{1}{2}$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$</p>	
---	--

<p>2. $\sin^2 x = 2\sin x$ $\sin^2 x - 2\sin x = 0$ $\sin x(\sin x - 2) = 0$ $\sin x = 0 \quad \sin x - 2 = 0$ $x = 0, \pi \quad \sin x = 2$ $x = 0, \pi$</p>	<p>3. $\csc^2 x - 2 = 0$ $\csc^2 x = 2$ $\csc x = \pm\sqrt{2}$ $\sin x = \frac{1}{\pm\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$</p>
--	--

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

4. $(\sqrt{3}\tan x - 1)(\tan x - \sqrt{3}) = 0$

$\sqrt{3}\tan x - 1 = 0$	$\tan x - \sqrt{3} = 0$
$\sqrt{3}\tan x = 1$	$\tan x = \sqrt{3}$
$\tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$x = \frac{\pi}{3}, \frac{4\pi}{3}$
$x = \frac{\pi}{6}, \frac{7\pi}{6}$	

$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}$

5. $2\sin^2 x + 3\cos x - 3 = 0$

$$2(1 - \cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$-2\cos^2 x + 3\cos x - 1 = 0$$

$\frac{-1}{-1}$	$\frac{-1}{-1}$	$\frac{-1}{-1}$	$\frac{-1}{-1}$
-----------------	-----------------	-----------------	-----------------

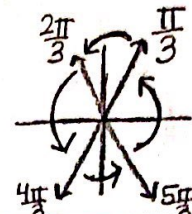
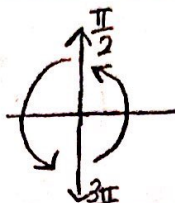
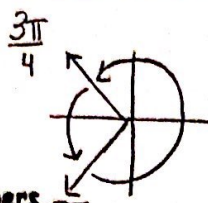
$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$2\cos x - 1 = 0$	$\cos x - 1 = 0$
$2\cos x = 1$	$\cos x = 1$
$\cos x = \frac{1}{2}$	$x = 0$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$x = 0, \frac{\pi}{3}, \frac{5\pi}{3}$



Solve for all real numbers.

6. $\cos x + \sqrt{2} = -\cos x + \cos x$
 $2\cos x + \sqrt{2} = 0$
 $2\cos x = -\sqrt{2}$
 $\cos x = -\frac{\sqrt{2}}{2}$
 $x = \frac{3\pi}{4} + 2\pi n$ *"every"*
 $x = \frac{5\pi}{4} + 2\pi n$

7. $\cot x \cos^2 x = 2\cot x$
 $\cot x \cos^2 x - 2\cot x = 0$
 $\cot x (\cos^2 x - 2) = 0$
 $\cot x = 0 \quad \cos^2 x - 2 = 0$
 $\cos^2 x = 2$
 $\cos x = \pm\sqrt{2}$
 $x = \frac{\pi}{2} + \pi n$

8. $4\sin^2 x - 3 = 0$
 $4\sin^2 x = 3$
 $\sin^2 x = \frac{3}{4}$
 $\sin x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$
 $x = \frac{\pi}{3} + \pi n$
 $x = \frac{2\pi}{3} + \pi n$

$2x + \sqrt{2} = 0$
 $2x = -\sqrt{2}$
 $x = -\frac{\sqrt{2}}{2}$

For functions of the form $\sin kx$: solve for (kx) , then divide by k .

Solve for all real numbers.

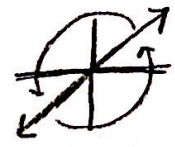
9. $\sin(2x) = -\frac{1}{2}$
 $\sin u = -\frac{1}{2}$
 $u = \frac{7\pi}{6} + 2\pi n$
 $u = \frac{11\pi}{6} + 2\pi n$
 $x = \frac{7\pi}{12} + \pi n$
 $x = \frac{11\pi}{12} + \pi n$

$u = 2x \Rightarrow \frac{u}{2} = x$



10. $\tan\left(\frac{x}{2}\right) - 1 = 0$
 $\tan u - 1 = 0$
 $\tan u = 1$
 $u = \frac{\pi}{4} + \pi n$

$u = \frac{x}{2} \Rightarrow 2u = x$



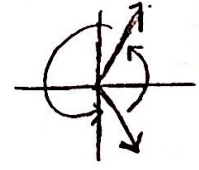
$x = \frac{\pi}{2} + 2\pi n$

11. $\cos 2x = -1$
 $\cos u = -1$
 $u = \pi + 2\pi n$
 $x = \frac{\pi}{2} + \pi n$

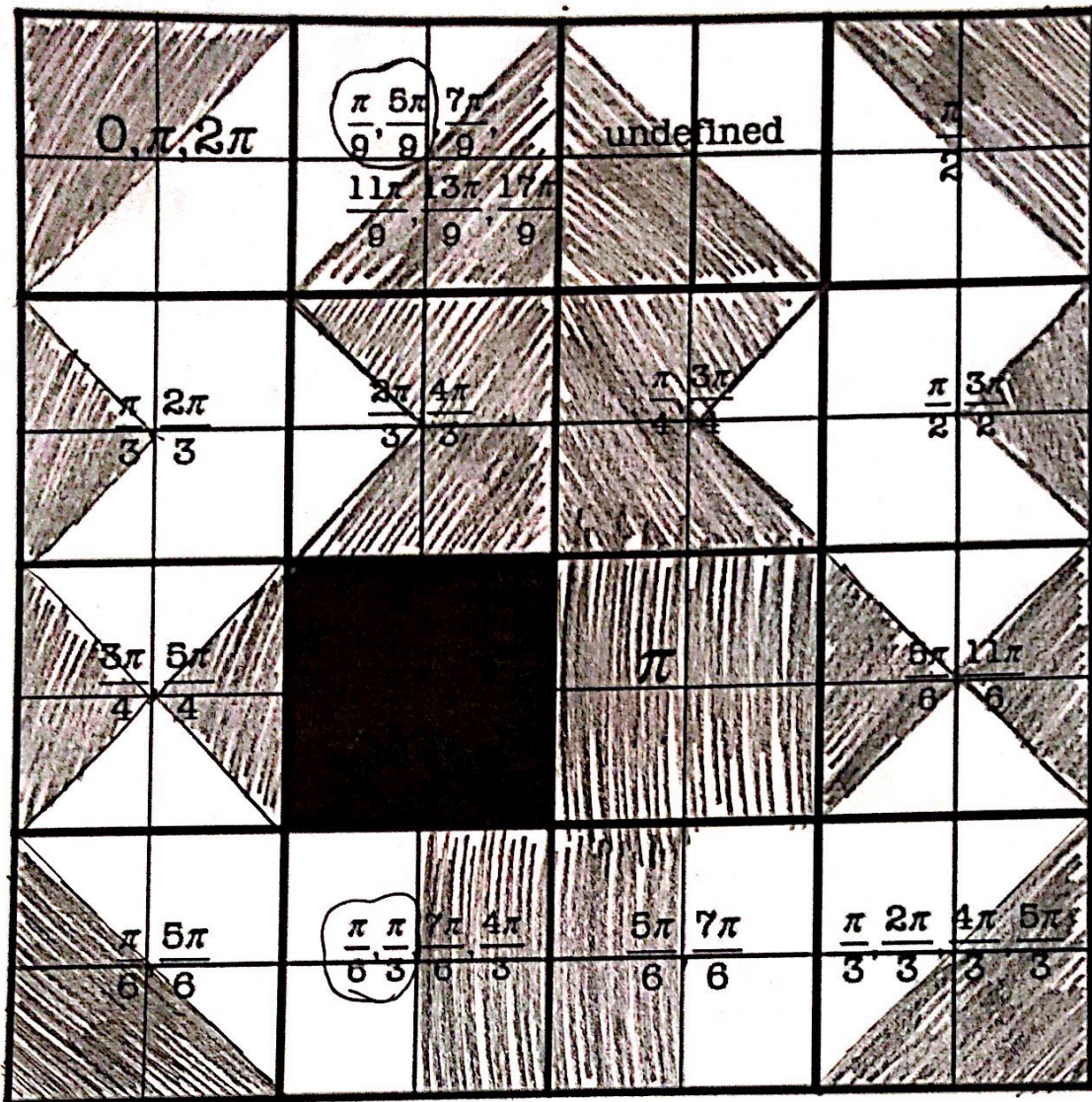
$u = 2x \Rightarrow \frac{u}{2} = x$

12. $\sec 4x = 2$
 $\sec u = 2$
 $\cos u = \frac{1}{2}$
 $u = \frac{\pi}{3} + 2\pi n$
 $u = \frac{5\pi}{3} + 2\pi n$

$u = 4x \Rightarrow \frac{u}{4} = x$



$x = \frac{\pi}{12} + \frac{\pi}{2} n$
 $x = \frac{5\pi}{12} + \frac{\pi}{2} n$



Solve each equation over the interval $[0, 2\pi]$ and then find the answers in the boxes above. Color the boxes according to the given pattern.

$\frac{\pi}{6}, \frac{5\pi}{6} \sin x = \frac{1}{2}$		$\frac{5\pi}{6}, \frac{7\pi}{6} \cos x = -\frac{\sqrt{3}}{2}$		$\frac{\pi}{4}, \frac{3\pi}{4} \sin x = \frac{\sqrt{2}}{2}$	
$\frac{3\pi}{4}, \frac{5\pi}{4} \cos x = -\frac{\sqrt{2}}{2}$		$0, \pi, 2\pi \tan x = 0$		$\pi \cos x = -1$	
$\frac{\pi}{2} \sin x = 1$		$\frac{5\pi}{6}, \frac{11\pi}{6} \tan x = -\frac{\sqrt{3}}{3}$		$\frac{\pi}{2}, \frac{3\pi}{2} \sin^2 x - 1 = 0$	
$\frac{\pi}{3}, \frac{2\pi}{3} \sin x = \frac{\sqrt{3}}{2}$		$\frac{2\pi}{3}, \frac{4\pi}{3} \cos x = -\frac{1}{2}$		$\frac{\pi}{6}, \frac{\pi}{3} \sin 2x = \frac{\sqrt{3}}{2}$	
$\frac{\pi}{4}, \frac{5\pi}{4} \cos 3x = \frac{1}{2}$		undefined $\sin x = 2$		$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \tan^2 x = 3$	