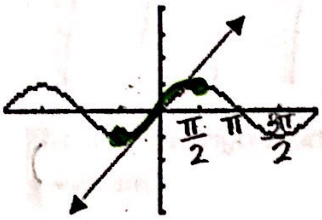
Name: ______________________________ Class Period: ______

Precalculus – Unit 2 Day 4
Inverse Trig Functions

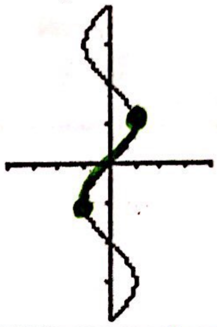
Definition of a **function**: (must pass the vertical line test) every element of the domain (x) is paired with exactly one element of the range (y).

Recall that the inverse of a function can be graphed by reflecting points over the $y = x$ line.



$y = \sin x$

Reflected over $y = x$



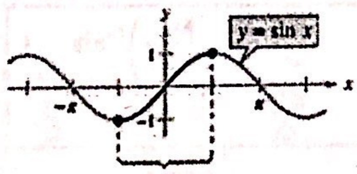
$y = \sin^{-1} x$

Is the reflected graph a function? Why or why not?

no; fails the VLT

In order to create a function, we must limit the domain. If the domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then the inverse will also be a function. Highlight the restricted domain on the graph above. The graph of $y = \sin^{-1} x$ will be a function as long as it has this restricted domain. By restricting the domain of each trigonometric function, we can create an inverse trigonometric function.

$y = \sin x$



Sin x has an inverse function on this interval.

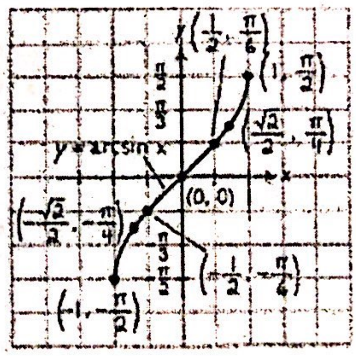
$(-\infty, \infty)$

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range: $[-1, 1]$

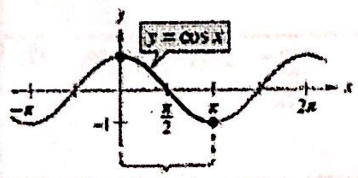
$y = \sin^{-1} x$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$y = \cos x$



Cos x has an inverse function on this interval.

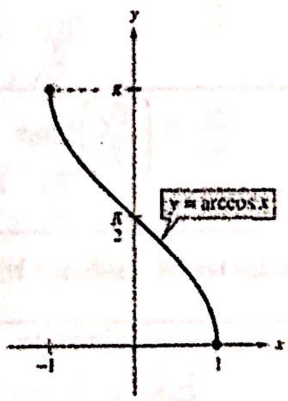
$(-\infty, \infty)$

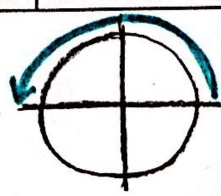
Domain: $[0, \pi]$ Range: $[-1, 1]$

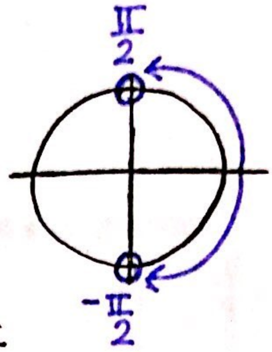
$y = \cos^{-1} x$

Domain: $[-1, 1]$

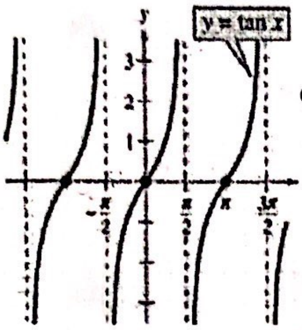
Range: $[0, \pi]$







$y = \tan x$



$(-\infty, \infty)$ except every $\frac{\pi}{2} \pm \pi$

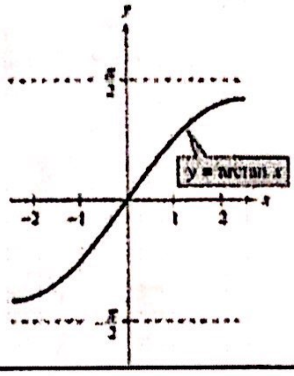
Domain:

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range:

$(-\infty, \infty)$

$y = \tan^{-1} x$



Domain:

$(-\infty, \infty)$

Range:

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The inverse trigonometric functions are denoted two ways:

$y = \arcsin x$		$y = \sin^{-1} x$
$y = \arccos x$	or	$y = \cos^{-1} x$
$y = \arctan x$		$y = \tan^{-1} x$

When evaluating an inverse trigonometric function such as the arcsine, remember that the "arcsine of x is the angle whose sine is x". You are determining the **ANGLE**. Also, you are only to give answers on the restricted ranges for each inverse trigonometric function. List these ranges below. *MEMORIZE these ranges!*

$y = \sin^{-1} x$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $y = \cos^{-1} x$ $[0, \pi]$ $y = \tan^{-1} x$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Examples: Find the exact value in radian measure without using a calculator.

at what angle is $\sin = -\frac{1}{2}$

1. $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\cancel{\frac{7\pi}{6}}, \cancel{\frac{11\pi}{6}}$	2. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ $\cancel{\frac{\pi}{4}}, \cancel{\frac{3\pi}{4}}$	3. $\sin^{-1}(2)$ undefined
4. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ $\frac{\pi}{6}, \cancel{\frac{11\pi}{6}}$	5. $\cos^{-1}(-5)$ undefined	6. $\cos^{-1}(-1) = \pi$
7. $\tan^{-1}(0) = 0$ $0, \cancel{\pi}$	8. $\tan^{-1}(-1) = -\frac{\pi}{4}$ $\cancel{\frac{3\pi}{4}}, \cancel{\frac{7\pi}{4}}$	9. $\arctan(\sqrt{3}) = \frac{\pi}{3}$ $\cancel{\frac{\pi}{3}}, \cancel{\frac{4\pi}{3}}$
10. $\arcsin(-1) = -\frac{\pi}{2}$ $\frac{3\pi}{2}$	11. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\frac{\pi}{3}, \cancel{\frac{5\pi}{3}}$	12. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\cancel{\frac{\pi}{6}}, \cancel{\frac{7\pi}{6}}$

Examples: Use a calculator to approximate the value in radian measure (if possible). Round values to the nearest ten-thousandth.

13. $\tan^{-1}(-8.45)$ ≈ -1.4530	14. $\arcsin(0.2447)$ ≈ 0.2472	15. $\arccos(2)$ undefined

Evaluate the given expression without the aid of a calculator.

1. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

2. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

3. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

4. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

5. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

6. $\arctan(1) = \frac{\pi}{4}$

7. $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

8. $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

9. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

11. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

12. $\tan^{-1}(-1) = -\frac{\pi}{4}$

13. $\sin^{-1} 0 = 0$

14. $\cos^{-1} 0 = \frac{\pi}{2}$

15. $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

16. $\arcsin(1) = \frac{\pi}{2}$

17. $\arccos(1) = 0$

18. $\tan^{-1} 0 = 0$

19. $\arcsin(-1) = -\frac{\pi}{2}$

20. $\arccos(-1) = \pi$

Find the exact value without a calculator.

21. $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

$\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

22. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

23. $\sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$

$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

24. $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$

$\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

25. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)=\frac{-\pi}{4}$

26. $\arccos\left(\sin\left(\frac{\pi}{3}\right)\right)$

$\arccos\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

27. $\sin\left(\tan^{-1}\left(\sqrt{3}\right)\right)$

$\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

28. $\cos\left(\tan^{-1}(-1)\right)$

$\cos\left(-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

29. $\tan^{-1}(\cos(\pi))$

$\tan^{-1}(-1)=\frac{-\pi}{4}$

Find an algebraic expression equivalent to the given expression.

omit

30. $\tan\left(\arccos\left(\frac{x}{3}\right)\right)$

31. $\sin(\arccos(x))$

32. $\sin\left(\arctan\left(\frac{x}{2}\right)\right)$

Evaluate using your calculator to find the approximate value. Express your answer in degrees.

33. $\sin^{-1}(.8621)$

59.55°

34. $\tan^{-1}(.5893)$

30.51°

35. $\cos^{-1}(-.3218)$

108.77°

36. arcsin (-.6821)

−43.01°

37. arctan (-1.6283)

−55.44°

38. arcos(.2814)

73.66°

Evaluate using your calculator to find the approximate value. Express your answer in radians

39. arcsin (.2618)

.265

40. $\cos^{-1}(-.8090)$

2.513

41. $\tan^{-1}(-1.7321)$

−1.047