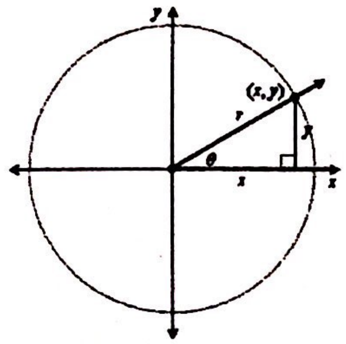
Name: ______________________________ Class Period: __________

Precalculus – Unit 1 Day 4

Trigonometric Functions of Any Angle

The trigonometric functions were originally restricted for $\boxed{\text{acute angles.}}$ These definitions can be extended to *any* angles by considering the standard position of an angle. Of course, if an angle is acute the new definitions should give the same result as the old definitions.

Acute Angle



Use the Pythagorean Theorem to find the length of the hypotenuse.
(Note that *r* is always $\boxed{\text{positive.}}$)

$$x^2 + y^2 = r^2$$

$$r = \sqrt{x^2 + y^2}$$

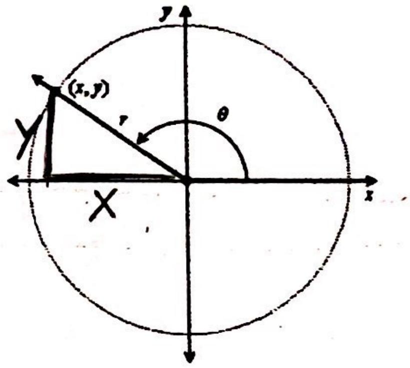
Definitions of Trignometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y}$

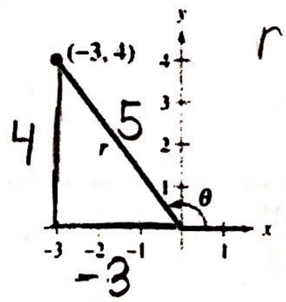
$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\qquad$ $\cot\theta = \dfrac{x}{y}$



Evaluate the exact values of the six trigonometric functions of the angle θ.

1.



$$r = \sqrt{(-3)^2 + (4)^2}$$

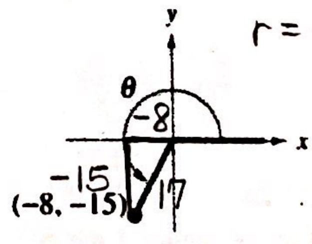
$$r = 5$$

$\sin\theta = \dfrac{4}{5}$ $\qquad$ $\csc\theta = \dfrac{5}{4}$

$\cos\theta = \dfrac{-3}{5}$ $\qquad$ $\sec\theta = \dfrac{5}{-3}$

$\tan\theta = \dfrac{4}{-3}$ $\qquad$ $\cot\theta = \dfrac{-3}{4}$

2.



$$r = \sqrt{(-8)^2 + (-15)^2}$$

$$r = 17$$

$\sin\theta = \dfrac{-15}{17}$ $\qquad$ $\csc\theta = \dfrac{17}{-15}$

$\cos\theta = \dfrac{-8}{17}$ $\qquad$ $\sec\theta = \dfrac{17}{-8}$

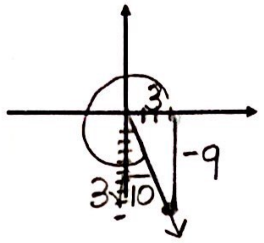
$\tan\theta = \dfrac{15}{8}$ $\qquad$ $\cot\theta = \dfrac{8}{15}$

The point given is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

3. (3,–9)

$r=\sqrt{(3)^2+(-9)^2}$

$r=\sqrt{90}=3\sqrt{10}$



$\sin\theta=\frac{-9}{3\sqrt{10}}=\frac{-3\sqrt{10}}{10}$ $\csc\theta=\frac{3\sqrt{10}}{-9}=\frac{\sqrt{10}}{-3}$

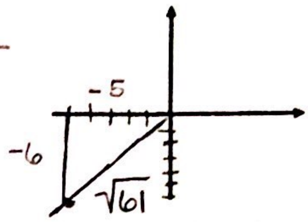
$\cos\theta=\frac{3}{3\sqrt{10}}=\frac{\sqrt{10}}{10}$ $\sec\theta=\frac{3\sqrt{10}}{3}=\sqrt{10}$

$\tan\theta=\frac{-9}{3}=-3$ $\cot\theta=\frac{3}{-9}=\frac{1}{-3}$

4. (–5,–6)

$r=\sqrt{(-5)^2+(-6)^2}$

$r=\sqrt{61}$



$\sin\theta=\frac{-6\sqrt{61}}{61}$ $\csc\theta=\frac{\sqrt{61}}{-6}$

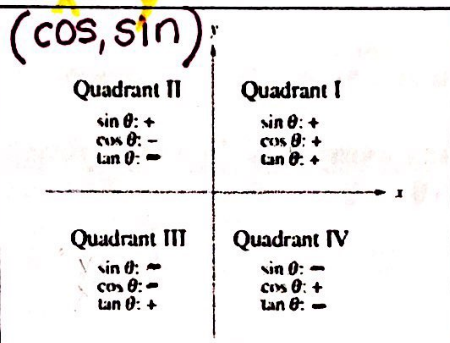
$\cos\theta=\frac{-5\sqrt{61}}{61}$ $\sec\theta=\frac{\sqrt{61}}{-5}$

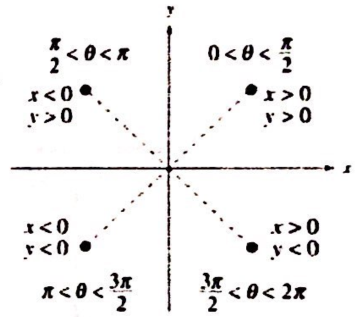
$\tan\theta=\frac{6}{5}$ $\cot\theta=\frac{5}{6}$

Recall the signs of the trigonometric functions in their four quadrants.

(remember that *r* is always positive)

x y
(cos, sin)





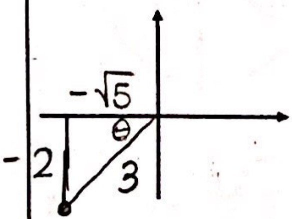
5. Given $\sin\theta=-\frac{2}{3}$ and $\tan\theta>0$, find $\cos\theta$ and $\cot\theta$.

$-2=y$

$3=r$

$x^2+(-2)^2=3^2$

$x=\sqrt{5}$



$\cos\theta=\frac{-\sqrt{5}}{3}$ $\cot\theta=\frac{\sqrt{5}}{2}$

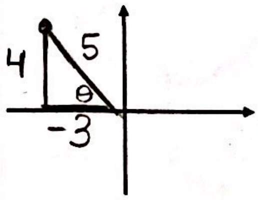
6. Given $\sin\theta=\frac{4}{5}$ and $\tan\theta<0$, find $\cos\theta$ and $\csc\theta$.

$4=y$

$5=r$

$x^2+4^2=5^2$

$x=3$



$\cos\theta=\frac{-3}{5}$ $\csc\theta=\frac{5}{4}$

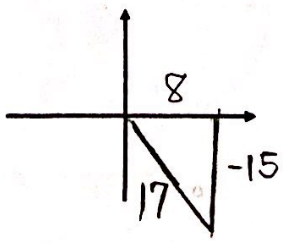
7. Given $\tan\theta=-\frac{15}{8}$ and $\sin\theta<0$, find the values of the six trigonometric functions of θ.

$-15=y$

$8=x$

$r=\sqrt{8^2+(-15)^2}$

$r=17$



$\sin\theta=\frac{-15}{17}$ $\csc\theta=\frac{17}{-15}$

$\cos\theta=\frac{8}{17}$ $\sec\theta=\frac{17}{8}$

$\cot\theta=\frac{8}{-15}$

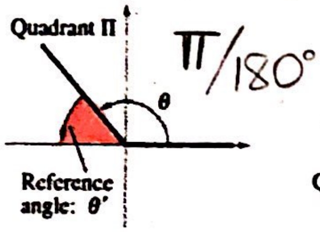
Reference Angles

The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at the corresponding acute angles called **reference angles**.

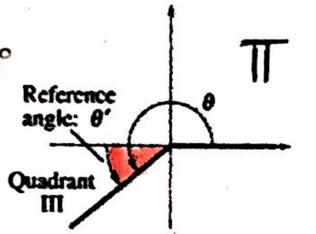
Definition of Reference Angle:

Let θ be an angle in Standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the x-axis.

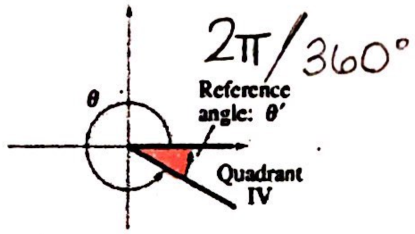
Note: reference angles are always $\boxed{\text{positive}}$.



$\theta' = \pi - \theta$ (radians)
$\theta' = 180° - \theta$ (degrees)



$\theta' = \theta - \pi$ (radians)
$\theta' = \theta - 180°$ (degrees)

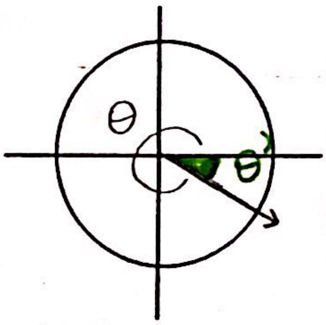


$\theta' = 2\pi - \theta$ (radians)
$\theta' = 360° - \theta$ (degrees)

Find the reference angle θ' and sketch θ and θ' in standard position.

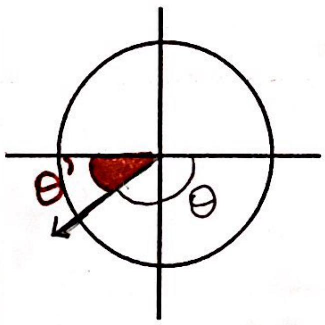
1. $\theta = 300°$

$\theta' = 60°$



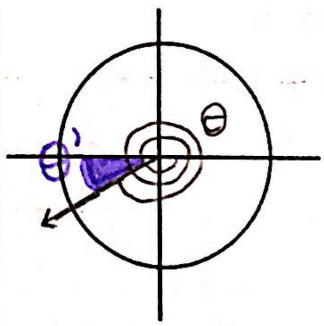
2. $\theta = -135°$

$\theta' = 45°$



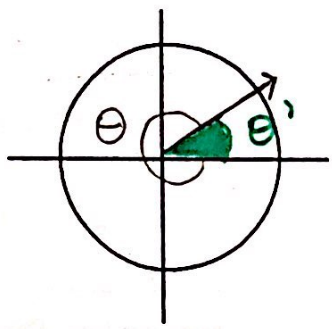
3. $\theta = -870° = -150°$

$\theta' = 30°$



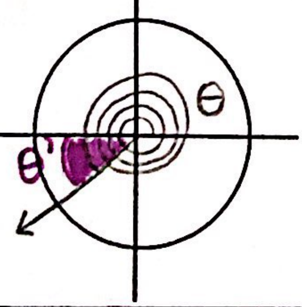
4. $\theta = -292°$

$\theta' = 68°$



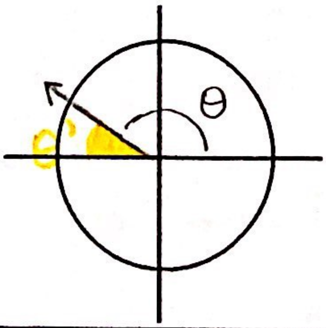
5. $\theta = \frac{51\pi}{7}$ $7\frac{2\pi}{7}$

$\theta' = \frac{2\pi}{7}$



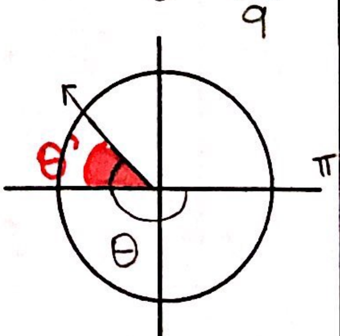
6. $\theta = \frac{4\pi}{5}$

$\theta' = \frac{\pi}{5}$



7. $\theta = -\frac{11\pi}{9}$

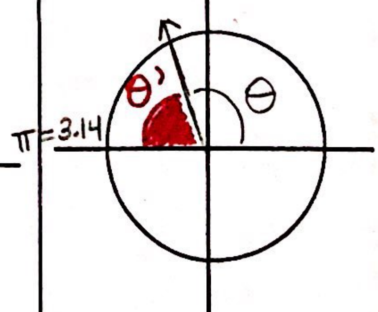
$\theta' = \frac{2\pi}{9}$



8. $\theta = 1.7$

$\theta' = 1.44$

$\frac{\pi}{2} = 1.5707$



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To find the value of a trigonometric function of any angle θ:

- Determine the function value for the associated reference angle θ'.
- Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

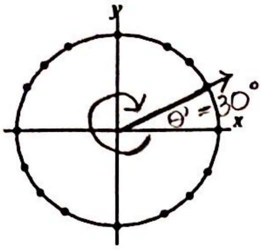
Evaluate the sine, cosine, and tangent of each angle without using a calculator.

9. $\theta = -330°$

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

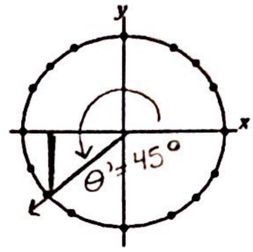


10. $\theta = 225°$

$\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan 45° = 1$

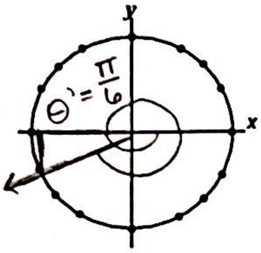


11. $\theta = \frac{-17\pi}{6}$

$\sin \frac{\pi}{6} = -\frac{1}{2}$

$\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

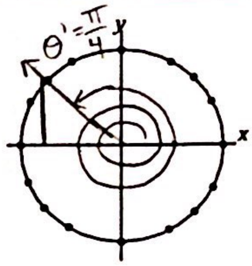


12. $\theta = \frac{19\pi}{4}$

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

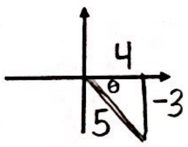
$\tan \frac{\pi}{4} = -1$



Find the indicated trigonometric value in the specified quadrant.

13. If $\sin\theta = -\frac{3}{5}$ and the angle is in quadrant IV, then find $\cos\theta$.

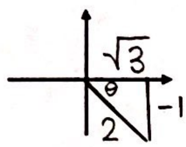
$\cos\theta = \frac{4}{5}$



14. If $\csc\theta = -2$ and the angle is in quadrant IV, then find $\cot\theta$.

$\sin\theta = \frac{-1}{2}$

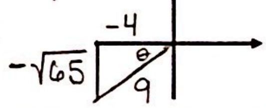
$\cot\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$



15. If $\sec\theta = -\frac{9}{4}$ and the angle is in quadrant III, then find $\tan\theta$.

$\cos\theta = -\frac{4}{9}$

$\tan\theta = \frac{-\sqrt{65}}{-4} = \frac{\sqrt{65}}{4}$



Find TWO solutions of the equation. Give you answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$. Do not use your calculator.

16. $\sin\theta = \frac{1}{2}$

$\theta = 30°$ or $\frac{\pi}{6}$

$\theta = 150°$ or $\frac{5\pi}{6}$

17. $\sin\theta = -\frac{1}{2}$

$\theta = 210°$ or $\frac{7\pi}{6}$

$\theta = 330°$ or $\frac{11\pi}{6}$

18. $\csc\theta = \frac{2\sqrt{3}}{3}$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60°$ or $\frac{\pi}{3}$

$\theta = 120°$ or $\frac{2\pi}{3}$

19. $\cot\theta = -1$ $\quad \tan\theta = -1$

$\theta = 135°$ or $\frac{3\pi}{4}$

$\theta = 315°$ or $\frac{7\pi}{4}$

20. $\sec\theta = -\frac{2\sqrt{3}}{3}$ $\quad \cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = 150°$ or $\frac{5\pi}{6}$

$\theta = 210°$ or $\frac{7\pi}{6}$

21. $\cos\theta = -\frac{1}{2}$

$\theta = 120°$ or $\frac{2\pi}{3}$

$\theta = 240°$ or $\frac{4\pi}{3}$